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Optimizing service-level and relevant cost for a stochastic multi-item cyclic production system

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ABSTRACT

A multi-item make-to-order production system in a stochastic environment is analyzed. Assuming a common cycle production approach, the impact of safety stock, cycle time, demand, processing time and setup time on service-level and total relevant cost (holding, setup and backorder cost) is determined. To illustrate this relationship a trajectory for the service-level with respect to the relevant cost (holding and setup) is presented. Furthermore algorithms to calculate the cycle time which leads to maximum service-level at constant safety stock and to calculate the pair cycle time and safety stock which minimize total relevant cost are introduced.

1. Introduction

Many manufacturing companies produce in common cycles. This means that in one common production cycle, for instance one month or one week, each product type is produced once in one lot. The impact of the common cycle length on inventory and holding cost is well known but the influence of the cycle time on service-level and safety stock which is needed to ensure a certain service-level is not widely known.

Ashayeri et al. (2006) point out the need for cyclic-production-inventory optimization models especially for the process industry. In their case study they show that a considerable cost reduction of about 30% can be realized by means of tailor-made cyclic planning. Bouchriha et al. (2007) study the optimization of setup and inventory cost for the paper machine in a pulp and paper mill. Here, production is also planned in common cycles with a predefined sequence due to technical restrictions. They define an optimum cycle length as well as the optimal batch size for a deterministic dynamic demand.

In this paper, which deals with a multi-item, stochastic, make-to-order production system with common production cycle, both the cost and the service-level are modeled as a function with respect to the cycle time and safety stock. Fluctuations in demand, processing times and setup times are taken into account. The objective criteria to evaluate cycle time and safety stock are based on holding cost, setup cost and service-level as well as penalty cost for backorders. The research objectives aim to:

(1) Determine the function describing the relationship between cycle time, safety stock and service-level
(2) Create a model to understand the impact of demand, processing time, setup time and number of product types on service-level and total relevant cost (inventory, setup and backorder cost)
(3) Find the pair cycle-time and safety stock which minimizes total relevant cost

This model enables some questions with practical relevance to be answered: Is it possible to improve service level as well as relevant cost by changing the number of product types? What is the influence of the variance of demand, processing time and setup time relating on service-level and cost structure?

The deterministic Economic Lot Scheduling Problem (ELSP) describes the situation for multi-items, dynamic demand and a general cycle policy. ELSP is NP-complete (see Hsu (1983)). Assuming a common cycle (in a fixed sequence the different product types are produced once during the common production cycle), Hanssmann (1962) calculates the optimal common cycle length which minimizes the holding and setup cost. The optimal production batch is determined by the optimal common cycle length multiplied by the demand rate.

Some paradoxes caused by high coefficient of variations of processing time or setup time can be observed, see Sarkar and Zangwill (1991) or more recently Samaddar and Hill (2007): WIP can go up for all products, even though setup time is reduced. Speeding up the processing rate does not necessarily result in a reduced number of products waiting to be processed. Cooper et al. (1998) analysis this anomaly and show that the

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Table 1
Definition of the parameters and variables.

Symbol	Description	Unit
d_i	Random variable describing the demand rate of product type *i*	N/H
μ_{d_i}	Expectation value of the demand rate of product type *i*	N/H
$\sigma^2_{d_i}$	Variance of the demand rate of product type *i*	N^2/H^2
τ_i	Random variable describing the processing time of product type *i*	H/N
μ_{τ_i}	Expectation value of the processing time of product type *i*	H/N
$\sigma^2_{\tau_i}$	Variance of the processing time of product type *i*	H^2/N^2
μ_{p_i}	Expectation value of the production rate of product type *i*	N/H
t_i	Random variable describing the setup time of product type *i*	H
μ_{t_i}	Expectation value of the setup time of product type *i*	H
$\sigma^2_{t_i}$	Variance of the setup of product type *i*	H^2
r_i	Safety stock of product type *i*	N
q_i	Lot-size of product type *i*	N
c_{H_i}	Unit holding cost of product type *i*	M/NH
c_{S_i}	Cost for one change-over to product type *i*	M
c_{B_i}	Cost for one unit of item *i* which is late (backorder cost for product type *i*)	M/N
t_c	Production cycle time	H
$t_{c,needed}$	Production cycle time needed to produce all lots	H
$\mu_{t_{c,needed}}$	Expectation value of the production cycle time needed	H
$\mu_{t_{missing}}$	Expectation value of the missing time to produce in time	H
T	Time period $[0,T]$	H
n	Number of different product types, $i=1, ..., n$	1
μ_{D_i}	Expectation value of the demand during the common cycle of product type *i*	N
$\sigma^2_{D_i}$	Variance of the demand during the common cycle of product type *i*	N^2

N is the number of product units; *M* is the money unit; *H* is the time unit.

expected waiting time can be minimized by adding a forced idle time. Samaddar and Whalen (2008) refine this idea by using a forced variable idle time in cyclic production systems.

Based on these insights it is useful and necessary to model the stochastic nature of a production system to ensure an acceptable approximation of the reality. Sox et al. (1999) provide a good survey and review of stochastic lot scheduling problems Table 1.

Federgruen and Katalan (1996) discuss the stochastic Economic Lot Scheduling Problem (SELSP) where several items are produced in one common capacitated production facility under stochastic demand, production times as well as setup times. The applied performance measure is the holding, backlogging and setup costs. The production strategies proposed are based on cyclical production schedules (each product type is produced once in one common cycle) and base-stock (production until the predefined base-stock level is reached or production of a batch size equal to the difference between base-stock level and the prevailing inventory level) whereby idle times are allowed. They develop an algorithm to evaluate the relevant performance measures and determine optimal strategies. Furthermore, they show that a deterministic treatment of a stochastic environment can result in costly backorders. Complementing their previous work, Federgruen and Katalan (1998) show how an effective periodic item-sequence can be selected.

Markowitz et al. (2000) consider the SELSP under the assumption of heavy traffic. They use dynamic cyclic policies and address two different objectives: the setup time problem (minimize the holding and backorder cost) and setup cost problem (minimize the holding, backorder and setup cost). A heavy traffic approximation is applied to make further progress with SELSP. This approach assumes that the server must be busy for the vast majority of time in order to meet average demand (this is equivalent to the customer required utilization nearing one).

A multiple item single stage production system with finite production rates and switchover times is considered in Anupindi and Tayur (1998). They focus on stochastic demand, stochastic inter arrival times and order focused measures (service-level and cost based on response times). They apply a modified periodic base-stock rule. In Anupindi et al. (1996) a non stationary demand and a stochastic lead time inventory problem are discussed. Treharne and Sox (2002) extend this to nonstationary demand with partial information.

Bourland and Yano (1997) combine safety stock with SELSP to balance setup cost, inventory cost and overtime cost. Saftey stocks are considered in the multiple-family ELSP model of Karalli and Flowers (2006) to address service-level. They discuss a normally distributed stationary demand problem with holding cost (for cycle stock and safety stock) and setup cost (for major setups between families and minor setups between items of the same family). The setup times as well as the processing times are assumed to be constant and deterministic. The applied safety stock is a function of the pre-specified service-level, the variance of the demand and the time interval between production runs for the item. They use a basic period approach allowing different cycles for different product types.

According to Sox et al. (1999) traditional approaches to stochastic lot scheduling problems in practice are independent stochastic control policies for instance the periodic-review order-up-to-level method (R,S) or the reorder-point method (Q,r). They also point out the lack of methods for determination of safety stock levels in models where the production and inventory control are constructed simultaneously. Brander and Forsberg (2006) present a method to define safety stocks and order-up-to levels for cyclic schedules and stochastic demand. Smits et al. (2004) use a fixed cyclic production sequence and a (R,S)-policy for the SELSP.

In this work the common cycle ELSP approach is applied combined with ideas of the stochastic control policy (Q,r). The lot-size of the (Q,r) policy is modeled by the product type independent cycle time and the reorder point is replaced by a safety stock, which depends on the variance of the demand to ensure the same service-level for every product type. To be able to manage the technical difficulties the expectation values and variances of several random variables are approximated and for the random variable demand rate the normal distribution is assumed to solve the governing equations.

The remainder is structured as follows. After defining the problem and its most important variables and parameters in

Section 2.1 based on the ELSP approach of Hanssmann (1962), the holding and setup cost are determined. In Section 2.2 the service-level is calculated by extending the ideas of the (Q,r) model. The total relevant cost is minimized in Section 2.3. The model in chapter 2 is followed by chapter 3 in which the model is numerically illustrated. In chapter 4 important managerial and theoretical implications are summarized. Chapter 5 is the conclusion.

2. Model

A multi-item model under stochastic demand, processing times as well as setup times is considered. It is assumed that in one production cycle all different product types are produced. This assumption conforms to the common cycle approach of ELSP, see Hanssmann (1962). For the changeovers, setup times and setup cost are taken into account. In addition to the setup cost, holding cost is modeled. For simplification, it is assumed that the service-level of different product types is identical. The main interest of this article is in the relationship between lot-sizes (or equivalent duration of the production cycle), cost (holding and setup) and service-level. The discussed service-level is the quantity oriented β-service level, that is the ratio number of items produced in time to number of all items.

Before formulating the model the variables and parameters are defined.

The statistical distribution function of a random variable x is denoted by F_x and its density function by f_x. Especially $F_{N(\mu;\sigma^2)}$ and $f_{N(\mu;\sigma^2)}$ denote the statistical distribution of the normal distribution with expectation value μ and variance σ^2.

Based on the common cycle approach, see Hanssmann (1962), the lot-size has to be equal to the average demand during the production cycle time.

$$q_i = t_c \mu_{d_i} \tag{1}$$

2.1. Cost

As for the stochastic reorder lot-sizing model (Q,r), see for instance Hopp and Spearman (1996), the randomness is neglected in the determination of the total holding and setup costs. The total inventory cost C_H for the period $[0,T]$ is determined by the safety stock and by half of the height of the triangle shaped inventory curve caused by the lot-sizing.

$$C_H = T\sum_{i=1}^{n} c_{H_i}\left(\frac{1}{2}\left(1-\frac{\mu_{d_i}}{\mu_{p_i}}\right)\mu_{d_i}t_c + r_i\right) \tag{2}$$

The mean processing rate is approximated by one over the expectation value of the processing time multiplied by a correction term depending on the coefficient of variation of the processing time, see Appendix Lemma 1.

$$\mu_{p_i} = \frac{1}{\mu_{\tau_i}}\left(1+\left(\frac{\sigma_{\tau_i}}{\mu_{\tau_i}}\right)^2\right) \tag{3}$$

The total change-over cost C_S for the period $[0,T]$ is determined by the number of setups needed.

$$C_S = T\sum_{i=1}^{n}\frac{1}{t_c}c_{S_i} \tag{4}$$

By differentiation of the total cost (inventory plus setup) with respect to the production cycle time the well-known cost-minimal ELSP-production common cycle time is gained, see Hanssmann (1962).

$$t_{C,\mathrm{ELSP}} = \sqrt{\frac{2\sum_{i=1}^{n} c_{S_i}}{\sum_{i=1}^{n} c_{H_i}(1-(\mu_{d_i}/\mu_{p_i}))\mu_{d_i}}} \tag{5}$$

The ELSP-production cycle time does not depend on the safety stock. The associated minimum costs with constant safety stocks are

$$C_{H,\mathrm{ELSP}} = T\sqrt{\frac{1}{2}\left(\sum_{i=1}^{n}\left(1-\frac{\mu_{d_i}}{\mu_{p_i}}\right)\mu_{d_i}c_{H_i}\right)\left(\sum_{i=1}^{n}c_{S_i}\right)} + T\sum_{i=1}^{n} r_i c_{H_i} \tag{6}$$

$$C_{S,\mathrm{ELSP}} = T\sqrt{\frac{1}{2}\left(\sum_{i=1}^{n}\left(1-\frac{\mu_{d_i}}{\mu_{p_i}}\right)\mu_{d_i}c_{H_i}\right)\left(\sum_{i=1}^{n}c_{S_i}\right)} \tag{7}$$

$$C_{\mathrm{ELSP}} = T\sqrt{2\left(\sum_{i=1}^{n}\left(1-\frac{\mu_{d_i}}{\mu_{p_i}}\right)\mu_{d_i}c_{H_i}\right)\left(\sum_{i=1}^{n}c_{S_i}\right)} + T\sum_{i=1}^{n} r_i c_{H_i} \tag{8}$$

For no safety stocks, the total holding cost is equal to the setup cost. In the general case the inventory cost is greater because of the safety stocks. The service-level is modeled in the next section.

2.2. Service-level

As a result of fluctuations, some due dates are not met. For a common cycle approach there are two main reasons for this.

(1) Random demand during the production cycle time is greater than the lot-size plus the safety stock. For very large lot-sizes or cycle times (the capacity is not used for customer required product) the risk of stock-outs caused by a higher demand during the cycle time than lot-size plus safety-stock will be higher than for small lot-sizes.
(2) Random required time to produce the batches is greater than the deterministic available time. For very small lot-sizes or cycle-times (a lot of changeovers have to be performed in a short cycle time) the risk of having insufficient capacity will be higher than for large lot-sizes.

Assuming statistically independent demand, the mean demand during the cycle time is defined by

$$\mu_{D_i} = t_c \mu_{d_i} \tag{9}$$

and the variance by

$$\sigma_{D_i}^2 = t_c \sigma_{d_i}^2 \tag{10}$$

For the service-level s_1, describing the effect that demand during the common production cycle time is greater than the produced lot-size plus the safety stock, the following identity holds true:

$$s_1 = F_{N(t_c\mu_{d_i},t_c\sigma_{d_i}^2)}(t_c\mu_{d_i}+r_i) = F_{N(0,1)}\left(\frac{r_i}{\sqrt{t_c}\sigma_{d_i}}\right) \underbrace{=}_{r_i=\alpha_1\sigma_{d_i}} F_{N(0,1)}\left(\frac{\alpha_1}{\sqrt{t_c}}\right) \tag{11}$$

Assuming that every product type has the same target service-level, the safety-stock is chosen by

$$r_i = \alpha_1 \sigma_{d_i} \tag{12}$$

This is why the average percentage of items which are not produced in time does not depend on the index i. A greater value α_1 increases the safety stocks and the product independent service-level s_1. If the safety stock is zero then the service-level s_1 is always equal to 0.5. If there is a strictly positive safety stock the service-level s_1 is between 0.5 and 1. For very small common production cycles or equally for very small lot-sizes the service-level s_1 converges to $1 = F_{N(0,1)}(\infty)$ and for very great lot-sizes to $0.5 = F_{N(0,1)}(0)$.

For short common production cycles an additional effect becomes more important and has to be considered. Short cycle times cause small production lots and more changeovers. Consequently, the time needed to produce and perform the setups has to be taken into account. The time needed in one common production cycle is defined by the time required for production (lot-size times the processing time) and by the time spent for the changeover activity.

$$t_{c,\text{needed}} = \sum_{i=1}^{n} t_c \mu_{d_i} \tau_i + t_i \tag{13}$$

The random variable $t_{c,\text{needed}}$ depends on the two random variables processing time τ_i and setup time t_i.

The time needed should be shorter than the production cycle time to ensure the production of all lots. Thus

$$t_{c,\text{needed}} = \sum_{i=1}^{n} t_c \mu_{d_i} \tau_i + t_i < t_c \tag{14}$$

should hold true. The value $t_c - t_{c,\text{needed}}$ can be interpreted as idle time, which is beneficial, particularly for highly variable setup times, according to Sarkar/Zangwill (1991); Federgruen/Katalan (1996); Cooper et al. (1998) or Samaddar/Whalen (2008). To enable an easier statistical handling of the requirement that the time needed has to be shorter than that which is available, the deterministic cycle time is isolated. This transformation of (14) yields to the equivalent inequality

$$x = \frac{\sum_{i=1}^{n} t_i}{1 - \sum_{i=1}^{n} \mu_{d_i} \tau_i} < t_c \tag{15}$$

The random variable x depends on the statistical distribution of the setup times and processing times and is independent of the cycle time. Inequality (15) can be interpreted as the setup time needed divided by the expression one minus customer required capacity measured in processing time per unit time has to be less than the cycle time. By applying some statistics for independent random variables, see Appendix Lemma 1, for the expectation value and variance of the random variable x the following identities are gained:

$$\mu_x = \frac{\sum_{i=1}^{n} \mu_{t_i}}{1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i}} \left(1 + \frac{\sum_{i=1}^{n} \mu_{d_i}^2 \sigma_{\tau_i}^2}{(1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i})^2} \right) \tag{16}$$

$$\sigma_x^2 = \frac{(\sum_{i=1}^{n} \mu_{t_i})^2}{(1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i})^2} \left(\frac{\sum_{i=1}^{n} \sigma_{t_i}^2}{(\sum_{i=1}^{n} \mu_{t_i})^2} + \frac{\sum_{i=1}^{n} \mu_{d_i}^2 \sigma_{\tau_i}^2}{(1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i})^2} \right) \tag{17}$$

To ensure feasibility in the deterministic case, the mean time needed x has to be less than the available time.

$$\mu_x < t_c \tag{18}$$

If the time needed is greater than the available cycle-time then not all items can be produced in time. The average missing time to produce all the production batches can be calculated by (see Appendix)

$$\mu_{t_{\text{missing}}} = \alpha_3 \int_{t_c}^{\infty} 1 - F_x(t) \mathrm{d}t$$

whereby

$$\alpha_3 = \frac{(1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i})^3}{(1 - \sum_{i=1}^{n} \mu_{d_i} \mu_{\tau_i})^2 + \sum_{i=1}^{n} \mu_{d_i}^2 \sigma_{\tau_i}^2} \tag{19}$$

The missing time leads to some reduction of the service-level. To model this behavior, weights g_i for the distribution of the missing time of the different product types i are introduced to ensure the same service-level for every product type.

$$\sum_{i=1}^{n} g_i = 1$$
$$g_i > 0 \tag{20}$$

Due to the missing time, the following average quantity is not produced in time:

$$\mu_{q_{i,\text{missing}}} = g_i \mu_{t_{\text{missing}}} \mu_{p_i} \tag{21}$$

Thus, putting all together, the service-level s is defined by

$$s = F_{N(t_c \mu_{d_i}, t_c \sigma_{d_i}^2)}(t_c \mu_{d_i} - g_i \mu_{t_{\text{missing}}} \mu_{p_i} + r_i)$$
$$= F_{N(0,1)}\left(\frac{r_i - g_i \mu_{t_{\text{missing}}} \mu_{p_i}}{\sqrt{t_c} \sigma_{d_i}} \right) = F_{N(0,1)}\left(\frac{\alpha_1 - \alpha_2 \mu_{t_{\text{missing}}}}{\sqrt{t_c}} \right) \tag{22}$$

whereby the product independent alpha-values and the weights are defined by:

$$\alpha_1 = \frac{r_i}{\sigma_{d_i}} \quad (\text{see}(12))$$
$$\alpha_2 = \frac{1}{\sum_{j=1}^{n} \frac{\sigma_{d_j}}{\mu_{p_j}}}$$
$$g_i = \alpha_2 \frac{\sigma_{d_i}}{\mu_{p_i}} \tag{23}$$

For the remainder it is important to take into account the fact that the average missing time is a function with respect to the common production cycle time. Furthermore, the product independent value α_1 determines the product type dependent safety stock to ensure the same service-level for every product type.

To find the cycle time which leads to maximum service-level the first derivative of the service-level with respect to the cycle time is determined and set to zero. The resulting equation for the cycle time maximizing the service-level is defined by (see Appendix):

$$2\alpha_2 \alpha_3 (1 - F_x(t_c)) t_c - \alpha_1 + \alpha_2 \alpha_3 \int_{t_c}^{\infty} 1 - F_x(t) \mathrm{d}t = 0 \tag{24}$$

Generally Eq. (24) is not analytically solvable. For the numerical solution a Newton approach is proposed. For one Newton step the following hold true, see Appendix:

$$t_{c,\text{new}} = t_{c,\text{old}} - \frac{h(t_{c,\text{old}})}{h'(t_{c,\text{old}})}$$
$$h(t_c) = 2\alpha_2 \alpha_3 (1 - F_x(t_c)) t_c - \alpha_1 + \alpha_2 \alpha_3 \int_{t_c}^{\infty} 1 - F_x(t) \mathrm{d}t$$
$$h'(t_c) = \alpha_2 \alpha_3 (-2 f_x(t_c) t_c - F_x(t_c) + 1) \tag{25}$$

In chapter 3 the proposed Newton method is applied and leads to fast convergence.

2.3. Minimizing total relevant cost

In this section the relevant cost, which consists of holding cost C_H, setup cost C_S and penalty cost for bad service-level C_B are considered and a model to minimize them is developed. The inventory cost is defined by (2). It is important to note that the safety stock r_i for each product type i depends on the factor α_1. The change-over cost is determined according to formula (4) and the service-level s is calculated by (22), whereby the expectation value of the time missing depends also on the cycle time. The penalty cost for bad service-level can be calculated as

$$C_B = (1 - s(t_c, \alpha_1)) T c_B \tag{26}$$

where the penalty cost for the backorders C_B equals the sum of the item individual calculative cost for one item whose due date is not met multiplied by average demand:

$$c_B = \sum_{i=1}^{n} c_{B_i} \mu_{d_i} \tag{27}$$

Assuming a linear cost structure the total relevant cost is described by

$$C(t_c,\alpha_1) = C_H(t_c,\alpha_1) + C_S(t_c) + C_B(t_c,\alpha_1) \tag{28}$$

Because of the common cycle approach and the assumption that all different product types should have the same service-level, the total relevant cost is a function with respect to the two decision variables cycle time t_c and factor α_1. The item independent cycle time determines the product type individual lot sizes, see formula (1) and the item independent factor α_1 determines the product type individual safety stocks, see formula (12).

By differentiating Eq. (28) with respect to the two decision parameters t_c and $\alpha_{1,}$ two equations defining the cost minimal pair cycle time and factor α_1 are gained. Before presenting the governing equations some abbreviations are introduced.

$$c_{HC} = \sum_{i=1}^{n} c_{H_i} \frac{1}{2}\left(1 - \frac{\mu_{d_i}}{\mu_{p_i}}\right)\mu_{d_i} \tag{29}$$

$$c_{HS} = \sum_{i=1}^{n} c_{H_i} \sigma_{d_i} \tag{30}$$

$$c_S = \sum_{i=1}^{n} c_{S_i} \tag{31}$$

$$\alpha(\alpha_1,t_c) = \frac{\alpha_1 - \alpha_2 \mu_{t_{\text{missing}}}(t_c)}{\sqrt{t_c}} \tag{32}$$

By using the cost abbreviations and (32), the total relevant cost (see (28)) with respect to t_c and α_1 can be written as:

$$C(t_c,\alpha_1) = T\left(c_{HC} t_c + \alpha_1 c_{HS} + \frac{c_S}{t_c} + (1 - F_{N(0,1)}(\alpha(t_c,\alpha_1)))c_B\right) \tag{33}$$

The four terms of the sum describe the four different associated costs, namely holding cost for the cycle stock, holding cost for the safety stock, setup cost and backorder cost. Differentiation yields to (see Appendix):

$$t_c = \left(\frac{f_{N(0,1)}(\alpha)c_B}{c_{HS}}\right)^2 \tag{34}$$

$$\alpha = 2\sqrt{t_c}\left(-\alpha_2\alpha_3(F_X(t_c) - 1) - \frac{c_{HC}}{c_{HS}} + \frac{c_S}{c_{HS} t_c^2}\right) \tag{35}$$

Similar to the classical (Q,r) approach, see for instance Hopp and Spearman (1996), these two equations for the unknown optimal cycle time and the auxiliary variable α have to be solved simultaneously. A two-dimensional Newton method is proposed:

(1) Select start values for the cycle time (for instance the ELSP cycle time according formula (5)) and for the factor α (for instance $\alpha = 1$)
(2) Calculate the two-dimensional function to be solved for zeros

$$f(t_c,\alpha) = \begin{pmatrix} t_c - (f_{N(0,1)}(\alpha)c_B/c_{HS})^2 \\ \alpha - 2\sqrt{t_c}(-\alpha_2\alpha_3(F_X(t_c) - 1) - (c_{HC}/c_{HS}) + (c_S/c_{HS} t_c^2)) \end{pmatrix} \tag{36}$$

(3) Calculate the Jacobi

$$J(t_c,\alpha) = \begin{pmatrix} 1 & \frac{-2c_B^2 f_{N(0,1)}(\alpha) f'_{N(0,1)}(\alpha)}{c_{HS}^2} \\ 2\alpha_2\alpha_3\left(\frac{1}{2\sqrt{t_c}}(F_X(t_c) - 1) + \sqrt{t_c} f_X(t_c)\right) + 3\frac{c_S}{c_{HS}\sqrt{t_c^5}} & 1 \end{pmatrix} \tag{37}$$

(4) Calculate the two-dimensional Newton change vector and determine better zeroes

$$h(t_{c,\text{old}},\alpha_{\text{old}}) = J^{-1}(t_{c,\text{old}},\alpha_{\text{old}}) f(t_{c,\text{old}},\alpha_{\text{old}})$$
$$\begin{pmatrix} t_{c,\text{new}} \\ \alpha_{\text{new}} \end{pmatrix} = \begin{pmatrix} t_{c,\text{old}} \\ \alpha_{\text{old}} \end{pmatrix} - h(t_{c,\text{old}},\alpha_{\text{old}}) \tag{38}$$

(5) Repeat Step (2) to (4) until the changes are sufficiently small and the zeroes are approximately found

$$\|h(t_{c,\text{old}},\alpha_{\text{old}})\| < \varepsilon \left\| \begin{pmatrix} t_{c,\text{old}} \\ \alpha_{\text{old}} \end{pmatrix} \right\| \wedge \|f(t_{c,\text{new}},\alpha_{\text{new}})\| < \varepsilon C_{\text{ELSP}} \tag{39}$$

(6) Calculate the optimal factor α_1 by applying (32)

$$\alpha_1 = \alpha\sqrt{t_c} + \alpha_2 \mu_{t_{\text{missing}}}(t_c) \tag{40}$$

Studies in the section numerical illustrations of the results show that this proposed algorithm generally converges quickly.

3. Numerical illustration of the results

The results are presented in two graphs shown in Fig. 1. In the first (see top left picture of Fig. 1) the two functions service-level as well as the sum of holding and setup cost are plotted with respect to the cycle time. The service-level with respect to the cycle time has one peak and there is a very steep slope for cycle time which is shorter than the cycle time which leads to the maximum service-level. This dramatic influence is caused by the fact that a short cycle time implies small lot-sizes and thus many changeovers, which need too much capacity. For very long cycle times the service-level converges to 0.5. The relevant cost (setup+holding) is a convex function with respect to the cycle time. In general, the cost-minimum cycle time is not equal to the service-level maximum cycle time. Note that this cost-minimum represents the minimal sum of setup and holding cost which differs from the minimal total cost calculated in Section 2.3. There penalty costs for backorder are also taken into account.

In the top right picture of Fig. 1 the service-level with respect to the relevant cost (setup+holding) is plotted as a parametric curve (parameter: t_c). Typically, this trajectory is a loop because generally there are two different service levels which can be reached with the same setup and holding cost (but different cycle times). This is illustrated in the two pictures at the bottom of Fig. 1. For a fixed level of setup and holding cost ($P1_x = P2_x$ in the left picture at the bottom) you get, in general, two different points on the cost function, and corresponding to the same cycle times, two values for the service level ($P1_y$ and $P2_y$). Now the right picture at the bottom of Fig. 1 can be constructed by plotting the setup and holding cost on the x-axis and the corresponding service level on the y-axis. As the setup and holding cost for $P1$ and $P2$ are the same, the two points are located on the same vertical line.

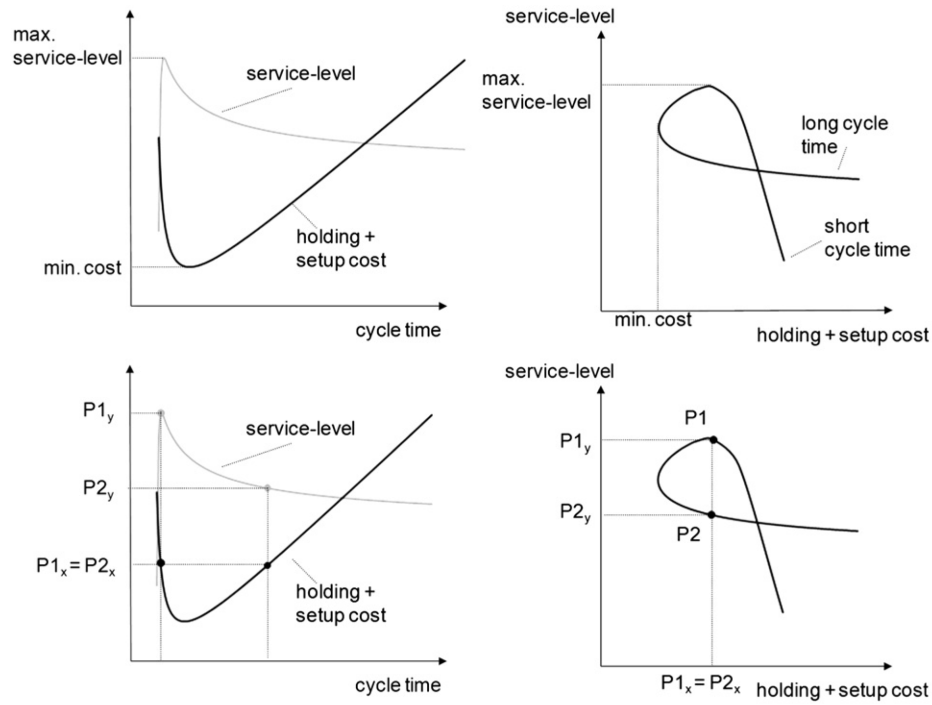

Fig. 1. Illustration of results.

Table 2
Definition of the basic setting A.

	μ_{d_i}	σ_{d_i}	μ_{τ_i}	σ_{τ_i}	μ_{t_i}	σ_{t_i}	c_{H_i}	c_{S_i}	c_{B_i}
P1	0.1	0.05	1	0.5	0.5	0.25	0.1	10	1
P2	0.2	0.1	0.6	0.3	1	0.5	0.5	20	5
P3	0.3	0.15	0.5	0.25	0.8	0.4	0.7	15	7

The left most point of the characteristic loop denotes the minimum cost point while the highest point indicates the maximum service-level point. Only cycle times in which curve points are between the cost minimum point and the service-level maximum point represent reasonable choices. For cycle times greater than the cost minimum one, a further increase of the cycle time worsens the service-level as well as the cost structure. For cycle times shorter than the service-level maximum one, a further decrease of the cycle time worsens the service-level as well as the cost structure. Consequently the total cost (setup, holding and penalty cost for bad service-level) minimum point has to be between the minimum cost point and the maximum service-level point.

For cycle time which is close to infinity, the service-level converges to 0.5 and for very small cycle-times the service-level goes to zero. Short cycle time means many setups and a high risk of not enough capacity. Long cycle time causes a high risk of stock-outs because the capacity is used for the wrong customer order. To ensure a certain service-level, a smaller lot-size enables smaller safety stock if enough capacity is available.

The slope of the trajectory at the cost minimum point is infinity. This means that a very small change in the employed cost (another cycle time) causes a high change in the service-level. Consequently, a shorter cycle time has a considerable impact on improving the service-level, while the cost will only increase a little. On the other side, the slope of the trajectory at the maximum service-level point is zero. This means that a cost change at this region has nearly no influence on the service-level.

Table 3
Definition of the comparison settings.

	Change
S0	Reference setting
S1	50%-Increase of the demand and demand deviation (coefficient of variation remains)
S2	50%-Increase of the coefficient of variation of demand
S3	50%-Increase of the processing time and processing time deviation
S4	50%-Increase of the coefficient of variation of processing time
S5	50%-Increase of the setup time and setup time deviation
S6	50%-Increase of the coefficient of variation of the setup time
S7	50%-Increase of safety-stock

3.1. Understanding the drivers on cost and service-level

This section demonstrates and discusses the impact of changes in the parameters on the cost and service-level. All the changes employed refer to the basic setting A with the following data.

Three product types *P*1, *P*2 and *P*3 are considered. According to Table 2 the coefficient of variation is 0.5 for the demand, processing times as well as for the setup times. For the safety stock factor α_1 the value 3 is assumed. Only one changed parameter is discussed in each case. The following settings are analysed and compared with the reference setting *S*0.

The same safety stock level is applied in settings *S*0 to *S*6. To ensure this in case *S*1 and *S*2 the safety stock factor α_1 is reduced to 2 (because the deviation is higher than in *S*0). The safety-stock increase in setting *S*7 is attained by the safety stock factor α_1 equal to 4.5 Table 3.

The next tables and figures illustrate the comparison.

Table 4 illustrates the results for the minimum cost (setup and holding cost) point and Table 5 for the maximum service-level point. The minimum holding and setup cost of the cases *S*0, *S*2, *S*5 and *S*6 are the same; this means that a change of setup times or of the coefficient of variation of the demand has no influence on the cost minimum point.

The most negative impact on the best attainable service-level is effected by the demand increase (*S*1), followed by an increase of demand fluctuation (*S*2), processing time increase (*S*3) and setup time increase (S5). This means that in practice a reduction of the demand fluctuation considerably improves the potential to increase the service-level.

In most of the cases (*S*1, *S*2, *S*3, *S*4, *S*5 and *S*6) the best reachable service-level is worse than in case *S*0 but the incurred cost (for the best service-level) is less than in case *S*0. Only in case *S*2 (higher demand fluctuation) for the service-level maximum point do both the service-level as well as the cost (holding and setup) decline.

Table 4
Numerical results for the cost (holding and setup) minimum point.

	*S*0	*S*1	*S*2	*S*3	*S*4	*S*5	*S*6	*S*7
min C_H+C_S								
t_c	17.79	15.00	17.79	18.37	17.57	17.79	17.79	17.79
C_H	30.10	34.80	30.10	29.29	30.41	30.10	30.10	32.50
C_S	25.30	30.00	25.30	24.49	25.61	25.30	25.30	25.30
C_H+C_S	55.39	64.79	55.39	53.78	56.02	55.39	55.39	57.79
C_B	7.63	14.53	10.17	7.74	7.59	7.63	7.63	4.58
C	63.02	79.33	65.56	61.53	63.61	63.02	63.02	62.37
s	0.76	0.70	0.68	0.76	0.76	0.76	0.76	0.86

Table 5
Numerical results for the service-level maximum point.

	*S*0	*S*1	*S*2	*S*3	*S*4	*S*5	*S*6	*S*7
max s min $C_{\text{inv}}+C_{\text{set}-\text{up}}$								
t_c	5.36	9.38	5.36	8.85	6.00	8.38	5.99	5.10
C_H	12.42	23.56	12.42	16.59	13.55	16.71	13.32	14.45
C_S	84.02	47.95	84.02	50.86	74.98	53.73	75.11	88.24
C_H+C_S	96.44	71.52	96.44	67.46	88.53	70.44	88.43	102.69
C_B	3.42	12.83	6.52	5.45	3.85	5.09	3.89	0.92
C	99.86	84.34	102.96	72.91	92.38	75.53	92.32	103.61
s	0.89	0.73	0.80	0.83	0.88	0.84	0.88	0.97

At the minimum cost (inventory and setup) point the service-levels in the cases *S*0, *S*3, *S*4, *S*5 and *S*6 are identical (there is no impact on the service-level at the cost minimum point by processing times as well as setup times). For shorter cycle times than of the cost minimum one the service-level depends on processing times as well as setup times.

In the following figures the bold curves describe the reference setting S0 and the fine curves correspond to the comparison setting Fig. 2.

The impact of demand and demand variation increase is a lower service-level and higher relevant cost for a given cycle-time. The cost minimum cycle time is shortened and the service-level maximum cycle time is larger. Furthermore, at the service-level maximum point the relevant cost is lower but the reachable service-level is worse than in the reference case S0. As in Markowitz (2000) lot-size grows with customer required capacity Fig. 3.

The increase of the demand variance causes no change in the cost curve but the service-level is considerably reduced for all cycle times. The cost minimum as well as the service-maximum cycle time remains unchanged Fig. 4.

An increase of the processing times reduces the cost for a fixed cycle time. Cooper et al. (1998) and Samaddar/Hill (2007) encounter a similar effect in a queuing system. To understand this surprising fact, the structure of the ELSP solution has to be considered. The demand and the cycle time determine the lot size. Both values are not changed in case *S*3. Consequently the lot sizes in case *S*0 and *S*3 are equal. However, in case *S*3 the processing times are longer than in *S*0. The time it takes to produce the production lot size is longer and the demand during the production period is higher than in case *S*0. Putting all together, the maximum inventory level in setting *S*3 is smaller than in case *S*0.

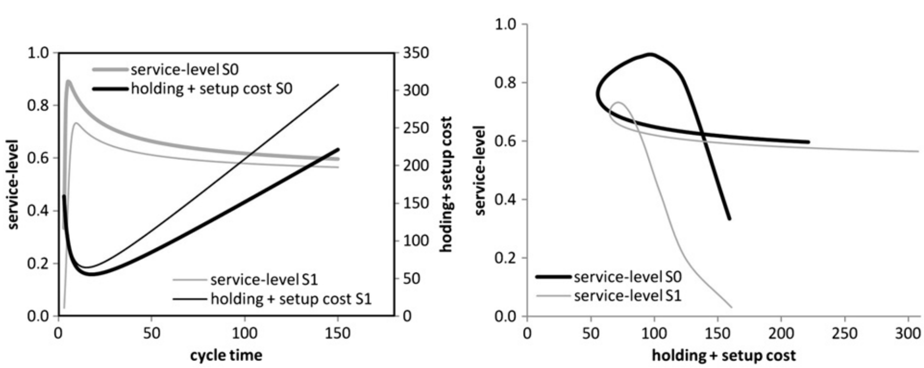

Fig. 2. Effect of demand and demand deviation increase.

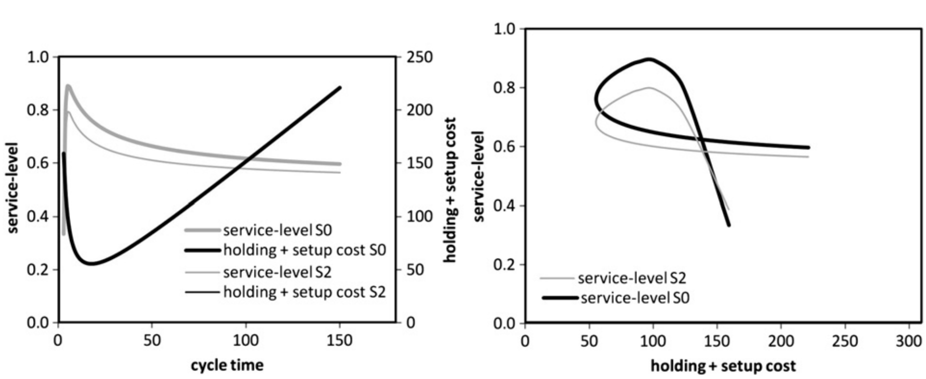

Fig. 3. Effect of coefficient of variation of demand increase.

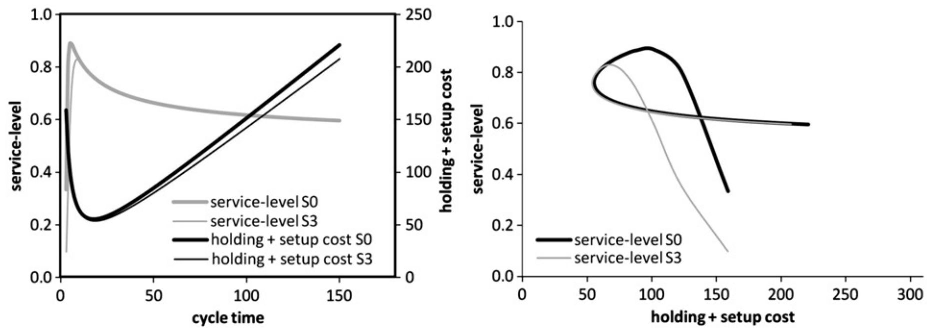

Fig. 4. Effect of processing time and processing time deviation increase.

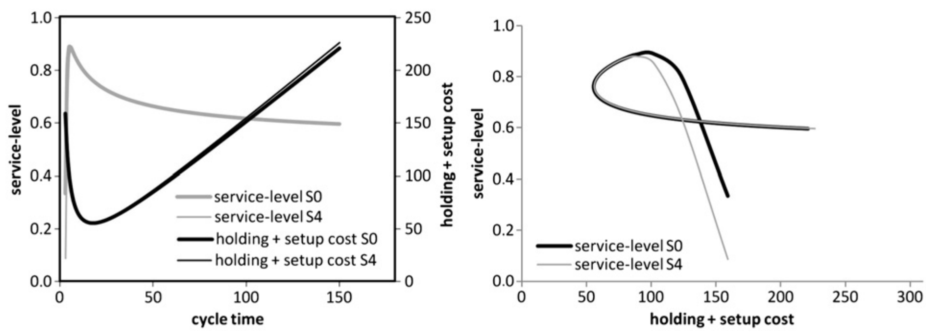

Fig. 5. Effect of coefficient of variation of processing time increase.

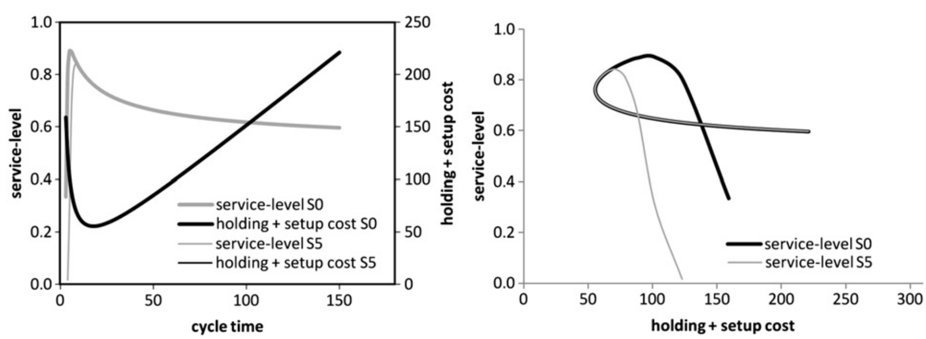

Fig. 6. Effect of setup time and setup time deviation increase.

This is why the cost (more accurately the holding cost) in *S*3 is lower than in case *S*0 for a fixed cycle time. The service-level, particularly near to the service-level maximum point, is worse in *S*3 than in *S*0. In the service-level curve with respect to the cost, relevant differences between case *S*0 and *S*1 occur only for small cycle times. Furthermore, the comparison of case *S*1 (higher demand) and case *S*3 (longer processing time) is of interest. In both cases the customer required cumulated processing time was increased by 50% but the effects are totally different. The holding cost in *S*1 increases because of larger lot-sizes but in *S*3 the holding cost decreases because more time is needed to produce the lot. The service-level in *S*1 is lower for all cycle times because of a reduced safety factor. In case *S*3 the service-level is worsened due to lack of capacity only for small cycle times Fig. 5.

On the left picture no big changes can be seen. However, the service-level curve with respect to the cost illustrates that for short cycle times, in case *S*4 the reachable service-level is lower than in case S0 for the same relevant cost (holding+setup). For long cycle times, there is no relevant difference between case *S*0 and *S*4 Fig. 6.

A setup time increase has no influence on the cost but the service-level is worsened for short cycle times. A reduction of setup time considerably improves the service-level for short cycle times (small lot sizes). This confirms the TPS-philosophy, see Ohno (1988), to apply small lot-sizes (preferable equal to one) by simultaneously ensuring very short setup times or, better , that no changeovers are needed. The cycle time which leads to maximum service-level is larger than in case *S*0. There is no relevant difference between case *S*0 and *S*5 for long cycle times Fig. 7.

Similarly to case *S*5 the service-level is reduced but the effect is somewhat smaller in setting *S*6 than *S*5. Again for long cycle times there is no relevant difference between case *S*0 and *S*6 Fig. 8.

The increase of safety stock considerably improves the service-level and increases the cost (holding cost).

3.2. Minimizing total relevant cost

The optimality condition is graphically illustrated before the presentation of the results concerning the pair cycle time and safety factor, which leads to a total cost minimum for the various cases Fig. 9.

The lower service-level curve with respect to the relevant cost illustrates case *S*0-, which is identical to case *S*0 except for safety

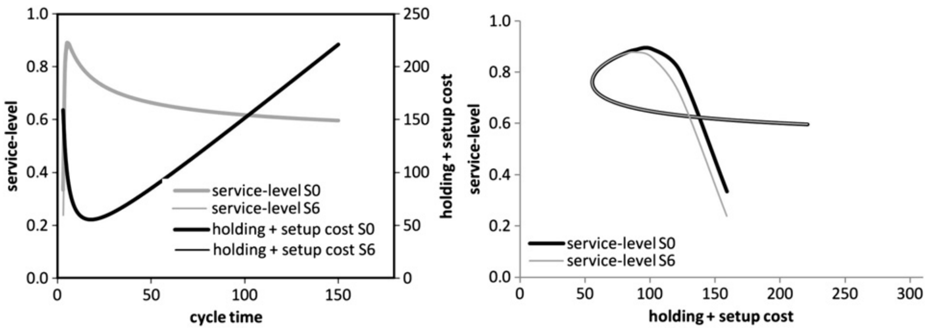

Fig. 7. Effect of coefficient of variation of setup time increase.

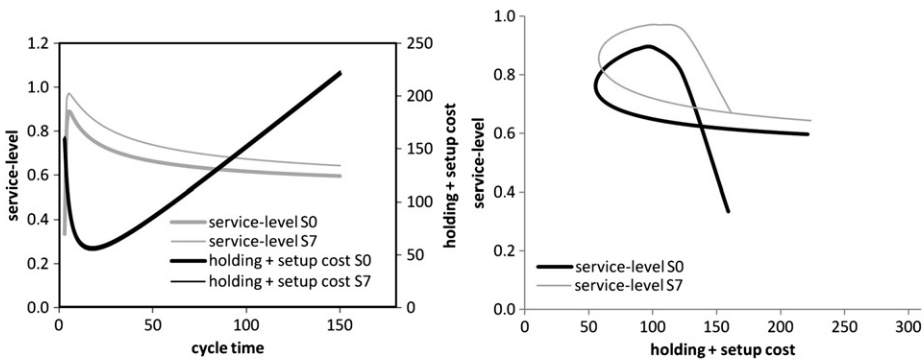

Fig. 8. Effect of safety factor increase.

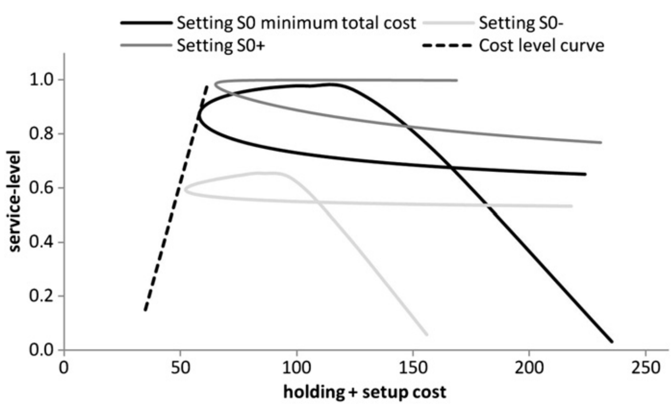

Fig. 9. Minimizing total cost.

Table 6
Numerical results for the total cost minimum problem.

	S0	*S1*	*S2*	*S3*	*S4*	*S5*	*S6*
min$C$							
t_c	16.51	13.69	16.58	17.01	16.32	16.51	16.51
C_H	31.03	38.39	30.72	30.25	31.34	31.03	31.03
C_S	27.25	32.87	27.15	26.46	27.57	27.25	27.25
C_H+C_S	58.29	71.25	57.86	56.71	58.90	58.29	58.29
C_B	3.93	5.16	7.43	4.01	3.89	3.93	3.93
C	62.21	76.42	65.30	60.72	62.80	62.21	62.21
s	0.88	0.89	0.77	0.87	0.88	0.88	0.88
α_1	4.72	4.59	2.98	4.74	4.71	4.72	4.72

stock factor $\alpha_1=1$. The uppermost curve illustrates case S0+ (safety factor equals 9). The total cost (holding+setup+backorder cost) minimum situation is demonstrated by the middle curve (optimal safety factor is equal to 4.72, see Tab. 5 last row). A higher safety stock improves the service-level (backorder cost is reduced) and increases the holding cost. The plotted straight line denotes the cost level curve with the lowest possible total cost. The tangent point corresponds to the minimum total cost point. The cost level curve with level c is defined by connecting all points which lead to the same total cost c (see Appendix):

$$s=\frac{C_H+C_S}{Tc_B}+1-\frac{c}{Tc_B} \tag{41}$$

Case *S*7 is not considered here because the optimal safety stock is determined to minimize total cost.

In setting *S*3 (increased processing times) the lowest total cost is reached. This surprising result can be explained by the fact that the inventory cost is decreased by increased processing times and a longer cycle time is chosen. Alltogether the setup as well as the holding cost decrease and this decrease is higher than the higher backorder cost. If processing time dependent cost, for instance labor cost, is incurred, then the total cost for *S*3 will be dramatically increased compared to setting S0.

The increase of the setup times or their deviation does not change the total minimum cost point because the changed setup times only have a relevant influence on cost and service-level for cycle times shorter than the cycle time which leads to total minimum cost.

For the cases *S*1 (higher demand) and *S*2 (higher demand deviation), a lower safety stock ensures minimum total cost. For the case *S*1 (higher demand) the cycle time is reduced to ensure minimum total cost.

3.3. Influence of number of product types

In this section the number of product types is doubled. To ensure a comparable capacity requirement the demand is halved. In detail the following data define case *S*8.

Six product types P1a, P2a and P3a and P1b, P2b and P3b are considered. According to Table 5 the coefficient of variation is 0.5 for the demand, processing times as well as for the setup times. For the safety stock factor α_1 the value 3 is assumed (the cumulated safety stock in both settings S0 and S8 are the same). The setup times or setup cost are not changed Tables 6–9.

The curves in Fig. 10 demonstrate that the cost increases and the service-level, mainly for short cycle times, decreases for double number of product types. The best attainable service-level decreases and more cost is incurred to achieve this situation. The detailed results of case S8 are given in the next two tables.

Table 7
Definition of setting S8.

	μ_{d_i}	σ_{d_i}	μ_{τ_i}	σ_{τ_i}	μ_{t_i}	σ_{t_i}	c_{H_i}	c_{S_i}	c_{B_i}
P1a	0.05	0.025	1	0.5	0.5	0.25	0.1	10	1
P2a	0.1	0.05	0.6	0.3	1	0.5	0.5	20	5
P3a	0.15	0.075	0.5	0.25	0.8	0.4	0.7	15	7
P1b	0.05	0.025	1	0.5	0.5	0.25	0.1	10	1
P2b	0.1	0.05	0.6	0.3	1	0.25	0.5	20	5
P3b	0.15	0.075	0.5	0.25	0.8	0.4	0.7	15	7

Table 8
Numerical results for minimal cost point.

	S0	S8
min C_H+C_S		
t_c	17.79	24.41
C_H	30.10	41.68
C_S	25.30	36.88
C_H+C_S	55.39	78.55
C_B	7.63	8.70
C	63.02	87.25
s	0.76	0.73

Table 9
Numerical results for maximal service-level point.

	S0	S8
max s		
t_c	5.89	10.12
C_H	13.17	20.10
C_S	76.42	88.89
C_H+C_S	89.59	108.99
C_B	3.70	5.75
C	93.30	114.74
s	0.88	0.82

4. Managerial and theoretical implications

This paper shows that for a common production cycle approach, the cycle length has a relevant influence on total relevant cost as well as on service-level. For too short cycle times, the total setup cost is high and the service-level decreases because of insufficient capacity. For too long cycle times, the total inventory cost is high and the service-level decreases because of not producing the customer required product type. Procedures to determine the cycle time which leads to maximum service-level and to calculate the optimal pair cycle time and safety stock which leads to minimum holding, setup and backorder cost are presented.

The advantage of the presented method is that lot-sizes and safety stocks of all product types can be easily determined by the Eqs. (1) and (12) assuming that every product type has the same cycle time and the same target service-level. The necessary results of the optimization procedure are only the two parameters cycle time and safety stock factor.

In practice, the following key findings, which are based on numerical experiments, can be interesting:

- Higher demand as well higher demand fluctuations reduce the service-level for each cycle time.
- Longer processing-times, longer setup times or their higher deviations worsen the service-level only for short cycle times.
- More safety-stock improves the service-level and increases the inventory cost for all cycle-times.
- Inventory cost increases for higher demand (especially for long cycle times).
- Inventory cost decreases for longer processing times (especially for long cycle times).
- A small reduction of the cycle time which leads to minimum holding and setup cost dramatically increases the service-level.
- The most efficient strategy to improve both total relevant cost and service-level is to reduce the demand fluctuations or the number of different product types to be produced.
- The most efficient measure to increase service-level is to increase safety stock, reduce demand fluctuations and in case of short cycle times to reduce setup times as well as processing times.
- The cycle time which maximizes service level is not the same as the one which minimizes the sum of holding and setup cost. In the tested numerical examples, the cycle time which maximizes service level is always shorter than the cost-minimizing cycle time.

Summing up the most important implications of this paper with practical relevance, the first contribution is the analysis of the influence of the number of product types on service level and cost structure. The numerical examples show that doubling the

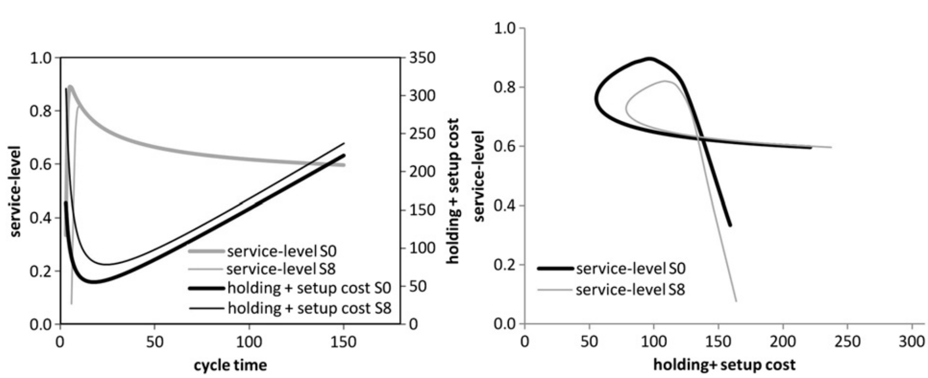

Fig. 10. Effect of doubling the number of product types.

number of product types worsens service level as well as the sum of holding and setup cost.

Another important input is that not only are the average values of demand, processing time and setup time relevant but that the variance of these values also play a decisive role. Reducing the variance of demand, processing time and setup time improves service level. Especially, the reduction of demand fluctuations offers a considerable potential for increasing service level.

From a theoretical point of view this work offers the following contributions:

- Not only is demand treated as a random variable, stochastic influences on processing times as well as on setup times are also included.
- In other works service-levels are used to determine safety stock levels (see Brander/Forsberg 2006). Here, an approach is presented which allows the optimization of service-level.
- Lot sizes and safety stocks are simultaneously determined.
- Unlike in other papers (Brander/Forsberg (2006); Smits et al. (2004)), where the SELSP is combined with a (*R*,*S*)-policy, here the (*Q*,*r*)-approach is used.

5. Conclusions

In this article, a multi-item stochastic production model with a common cycle approach is considered. Based on the developed analytical formula describing the relationship between service-level, safety stock and cycle time, the impact of the average values as well as the deviations of demand, processing time and setup time on service-level and cost (holding, setup and backorder) are demonstrated. To illustrate the results, a trajectory describing the service-level with respect to the cost (holding and setup cost) is put forward. Furthermore, algorithms to determine the maximum service-level point as well as the minimum total relevant cost point are presented.

In further research, more stochastic elements (yield loss, machine breakdowns) should be included in the model and the common cycle approach should be extended to more general dynamic periodic policies (see for instance Karalli/Flowers (2006)). A further useful extension is to take into account the stochastic nature of the problem in calculating the inventory and setup cost.

Appendix

Lemma 1. For the expectation value and variance of the ratio of the two random variables x und y the following identities hold approximately true:

$$\mu_{x/y} = \frac{\mu_x}{\mu_y}\left(1+\left(\frac{\sigma_y}{\mu_y}\right)^2\right)\sigma^2_{x/y} = \frac{\mu_x^2}{\mu_y^2}\left(\left(\frac{\sigma_x}{\mu_x}\right)^2+\left(\frac{\sigma_y}{\mu_y}\right)^2\right) \tag{42}$$

Furthermore, for the random variable one over x the expectation value can be approximated by:

$$\mu_{1/y} = \frac{1}{\mu_y}\left(1+\left(\frac{\sigma_y}{\mu_y}\right)^2\right) \tag{43}$$

Proof. See Wilrich and Henning (1987). □

Determination of the average missing time

$t_{c,\text{needed}} = t_c + t_{\text{missing}}$

(only positive values of t_{missing} are missing times)

$$\begin{aligned} &t_c\sum_{i=1}^{n}\mu_{d_i}\tau_i - t_c + \sum_{i=1}^{n} t_i = t_{\text{missing}} \\ &\frac{\sum_{i=1}^n t_i}{1-\sum_{i=1}^n \mu_{d_i}\tau_i} - t_c = \frac{t_{\text{missing}}}{1-\sum_{i=1}^n \mu_{d_i}\tau_i} = \bar{t}_{\text{missing}} \\ &x - t_c = \bar{t}_{\text{missing}} \end{aligned} \tag{44}$$

$$\mu_{\bar{t}_{\text{missing}}} = \int_{t_c}^{\infty} f_x(t)(t-t_c)\mathrm{d}t = \int_{t_c}^{\infty} 1-F_x(t)\mathrm{d}t \tag{45}$$

$$\begin{aligned} \mu_{\bar{t}_{\text{missing}}} &= \frac{\mu_{t_{\text{missing}}}}{1-\sum_{i=1}^n \mu_{d_i}\mu_{\tau_i}}\left(1+\frac{\sum_{i=1}^n \mu_{d_i}^2\sigma_{\tau_i}^2}{(1-\sum_{i=1}^n \mu_{d_i}\mu_{\tau_i})^2}\right) \\ \mu_{t_{\text{missing}}} &= \frac{(1-\sum_{i=1}^n \mu_{d_i}\mu_{\tau_i})^3 \int_{t_c}^{\infty} 1-F_x(t)\mathrm{d}t}{(1-\sum_{i=1}^n \mu_{d_i}\mu_{\tau_i})^2 + \sum_{i=1}^{n}\mu_{d_i}^2\sigma_{\tau_i}^2} = \alpha_3\int_{t_c}^{\infty} 1-F_x(t)\mathrm{d}t \end{aligned} \tag{46}$$

Equation for the determination of the cycle time which maximizes the service-level

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t_c}s(t_c) &= f_{N(0,1)}\left(\frac{\alpha_1-\alpha_2\mu_{t_{\text{missing}}}}{\sqrt{t_c}}\right) \\ &\quad\times\frac{-\alpha_2(\mathrm{d}/\mathrm{d}t_c)\mu_{t_{\text{missing}}}(t_c)\sqrt{t_c}-(\alpha_1-\alpha_2\mu_{t_{\text{missing}}})\frac{1}{2\sqrt{t_c}}}{t_c} = 0 \\ &-2\alpha_2\frac{\mathrm{d}}{\mathrm{d}t_c}\mu_{t_{\text{missing}}}(t_c)t_c-\alpha_1+\alpha_2\mu_{t_{\text{missing}}}(t_c) = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t_c}\mu_{\bar{t}_{\text{missing}}}(t_c) = \frac{\mathrm{d}}{\mathrm{d}t_c}\int_{t_c}^{\infty} 1-F_x(t)\mathrm{d}t = -1+F_x(t_c) \\ &2\alpha_2\alpha_3(1-F_x(t_c))t_c-\alpha_1+\alpha_2\alpha_3\int_{t_c}^{\infty} 1-F_x(t)\mathrm{d}t = 0 \end{aligned} \tag{48}$$

Derivative of the left side of Eq. (24)

$$\begin{aligned} h(t_c) &= 2\alpha_2\alpha_3(1-F_x(t_c))t_c-\alpha_1+\alpha_2\alpha_3\int_{t_c}^{\infty} 1-F_x(t)dt = 0 \\ h'(t_c) &= \frac{d}{dt_c}h(t_c) = 2\alpha_2\alpha_3(1-f_x(t_c)t_c-F_x(t_c))+\alpha_2\alpha_3F_x(t_c)-\alpha_2\alpha_3 \\ &= \alpha_2\alpha_3(-2f_x(t_c)t_c-F_x(t_c)+1) \end{aligned} \tag{49}$$

Equations for the determination of the optimal cycle time and factor α_1 which minimize the total relevant cost

$$\begin{aligned} \frac{d}{dt_c}C(t_c,\alpha_1) &= T\left(c_{HC}-\frac{c_S}{t_c^2}-\left(\frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}}(-\alpha_2\alpha_3(F_x(t_c)-1)\right.\right. \\ &\quad\left.\left.-\frac{1}{2t_c}\left(\alpha_1-\alpha_2\alpha_3\int_{t_c}^{\infty}1-F_x(t)dt\right)\right)c_B\right) \\ &= T\left(c_{HC}-\frac{c_S}{t_c^2}-\left(\frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}}\left(-\alpha_2\alpha_3(F_x(t_c)-1)-\frac{\alpha}{2\sqrt{t_c}}\right)\right)c_B\right) = 0 \\ &\Rightarrow c_{HC}-\frac{c_S}{t_c^2}-\left(\frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}}\left(-\alpha_2\alpha_3(F_x(t_c)-1)-\frac{\alpha}{2\sqrt{t_c}}\right)\right)c_B = 0 \end{aligned} \tag{50}$$

$$\begin{aligned} \frac{d}{d\alpha_1}C(t_c,\alpha_1) &= T\left(c_{HS}-\frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}}c_B\right) = 0 \Rightarrow c_{HS}-\frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}}c_B = 0 \\ &\Rightarrow \frac{f_{N(0,1)}(\alpha)}{\sqrt{t_c}} = \frac{c_{HS}}{c_B} \Rightarrow t_c = \left(\frac{f_{N(0,1)}(\alpha)c_B}{c_{HS}}\right)^2 \end{aligned} \tag{51}$$

$$\begin{aligned} &\Rightarrow c_{HC}-\frac{c_S}{t_c^2}-\left(\frac{c_{HS}}{c_B}\left(-\alpha_2\alpha_3(F_X(t_c)-1)-\frac{\alpha}{2\sqrt{t_c}}\right)\right)c_B = 0 \\ &\Rightarrow \alpha = 2\sqrt{t_c}\left(-\alpha_2\alpha_3(F_X(t_c)-1)-\frac{c_{HC}}{c_{HS}}+\frac{c_S}{c_{HS}t_c^2}\right) \end{aligned} \tag{52}$$

Cost level curves

$$c = C_H + C_S + (1-s)Tc_B \Rightarrow s = \frac{C_H + C_S}{Tc_B} + 1 - \frac{c}{Tc_B} \quad (53)$$

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