Two Measures of Production System Flexibility and their Application to Identify Optimal Capacity Investment

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Abstract

We motivate the use of a company's capability to handle the requirements concerning *mean customer required lead time* and *customer demand variation* as measures for production system flexibility within a Value Chain. Thus, the customer driven production planning concept is applied which links together customer demand variation, mean customer required lead time and service level. Thereby inventory holding and capacity costs for certain levels of required *customer demand variation* and *mean customer required lead time* are derived. For a service level constraint model, the cost optimum for capacity and inventory holding costs is shown to imply the ability to handle a certain *mean customer required lead time* flexibility when *capacity demand variation* is predefined. Further, the optimal costs are shown to increase in capacity demand variance. Applying the developed method a manager can either identify the minimum *mean customer required lead time* for a predefined *capacity demand variation* or he/she can identify the optimal values for the two flexibility measures discussed to minimize costs.

Keywords: manufacturing, flexibility management, customer driven production planning, inventory, capacity investment analysis, Value Chain demand variation

1. Introduction

Flexibility can be defined as the ability to react to changes in specification, amount and due date of customer orders, given a certain system specification. All these measures can be transformed into changes in the customer required lead time

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and to the customer demand variation. Therefore, the two flexibility measures for a customer – supplier relationship within a Value Chain can be identified as:

- the ability to handle short mean customer required lead times and
- the ability to handle a certain level of customer demand variation.

Though both measures are linked to local production facilities, they are of major importance for the overall Value Chain performance in two respects: First, any production site across the whole value creation process may be affected heavily by respective demand variance and thus negatively influence the entire Value Chain performance due to limited performance and flexibility. Second, suchlike demand fluctuations can exponentially amplify across several Value Chain stages (also known as "bull-whip" effect, cp. Forrester (1961), Lee et al. (1997)). This effect can be unfavorably enforced through an insufficient local handling of the balance between customer required lead time and associated key indicators.

The customer required lead time is defined as the time between when an order is stated (or restated if changes occur) and the requested due date (see Jodlbauer and Altendorfer (2010) or Altendorfer (2010)). Even though there already exists a large set of literature on flexibility measures as well as on their influence on the company performance, a huge proportion of this literature is of a qualitative nature. These papers discuss flexibility as a rather generic and merely measurable construct that needs to be further operationalized (frequently cited definitions are provided e.g. in Ackoff (1971), Sethi and Sethi (1990), Upton (1994) and, more recently, Zhang et al. (2003) or Reichhart and Holweg (2007)). Most of the quantitative approaches available, limit their scope of application to a very specific context or research question (see e.g. Das and Abdel-Marek (2003), Ahlert et al. (2009), Francas et al. (2011) or Palominos et al. (2009)). Currently available flexibility literature is extended in this paper by developing and applying a set of equations concerning the inherent relationship between the two aforementioned general flexibility measures. The modeling frame applied is a single-stage production system whereby this single stage can either be a single machine, a set of machines or a whole production plant. The main finding is that for any customer demand variation a certain lower bound for the mean customer required lead time is already implied when minimizing capacity and inventory costs with respect to a service level constraint. The practical implication is that whenever the downstream company in a Value Chain requires a certain demand flexibility from its upstream supplier due to its order variation, it also implicitly defines the mean customer required lead time flexibility that can be provided by this supplier (if the supplier does not change its capacity). Studying the relationship between these two flexibility measures contributes to a better understanding of Value Chain coherences occurring due to flexibility requirements, including performance and cost issues.

The Paper is structured as follows: Section 2 reviews the relevant literature on flexibility measures and production logistics related key figures. In Section 3 the relationship between mean customer required lead time flexibility and customer demand variation flexibility is developed. A numerical study is provided in Section 4 and we conclude in Section 5.

2. Literature review

The literature linked to the research topic of this paper can be split up into flexibility-related issues and contributions on production logistical key figures. Especially literature using the two flexibility measures applied in this paper is reviewed in the first subsection. The link between these two flexibility measures from a production logistical point of view is reviewed in the second subsection.

2.1 Definitions and application of flexibility

The term “flexibility” is discussed heterogeneously in the relevant Supply Chain literature (see Stevenson and Spring (2007) for an overview). A high amount of contributions provide rather generic definitions: For instance Zelenovic (1982) describes the flexibility of a production system as the capacity to adapt to changing environmental conditions and process requirements. Upton (1994) defines flexibility as the ability to change or react with little penalty and Reichhart and Holweg (2007) investigate several previously proposed flexibility definitions and summarize that flexibility is the ability of a system to adapt to internal or external influences, thereby acting or responding to achieve a desired outcome.

A more tangible, though, still qualitative perspective is applied in other publications that primarily apply a manufacturing-related point of view. These contributions are mostly compatible with the aforementioned generic definitions, although, according to Palominos et al. (2009), the advantage of high compatibility is accompanied by a lack of operational applicability, at least within a manufacturing context. For instance Olhager et al. (2001) discuss the application of chase strategies to achieve flexibility and adaptability in make-to-order (MTO) settings. They link manufacturing issues with demand-related aspects. Sethi and Sethi (1990) are further specifying flexibility as the ability to make different parts without major setups. In addition to the heterogeneous flexibility discussion, other terms like responsiveness (see e.g. Reichhart and Holweg (2007)) and agility (see e.g. Bernardes and Hanna (2009) and Narasimhan et al. (2006)) are applied to describe constructs that are similar (or corresponding) to flexibility.

Tab. 1. Relevant publications on flexibility.

Source	focus	lead time	demand variation
Tang and Tomlin (2008)	flexibility as a means for Supply Chain risk mitigation in disruptive environments (quantitative approach)	–	–
Salvador et al. (2007)	mix and volume flexibility in MTO settings (empirical investigation)	–	implicitly implied
Özbayrak et al. (2006)	investigate MTO control policies with fluctuating demand (simulation model)	explicitly used	explicitly used
Merzifonluoğlu and Geunes (2006)	determination of optimum demand, production and inventory levels (quantitative approach)	implicitly implied	explicitly used
Moattar Husseini et al. (2006)	quantitative analysis of the relation between resource level and output flexibility in a Just-in-Time environment	implicitly implied	explicitly used
Bish et al. (2005)	hedging flexible capacity against forecast errors in MTO settings while keeping swings in production low	implicitly implied	explicitly used
Zhang et al. (2005)	impacts of logistical flexibility on customer satisfaction (empirical evidence)	implicitly implied	implicitly implied
Wijngaard (2004)	influence of advance demand information on production and inventory control (quantitative approach)	explicitly used	explicitly used
Zhang et al. (2003)	manufacturing flexibility as critical issue in Value Chains (empirical evidence)	implicitly implied	explicitly used
Garavelli (2003)	product flow optimization in Supply Chains without dramatically increasing flexibility costs (simulation model)	explicitly used	explicitly used
Oke (2003a)	drivers of volume flexibility in manufacturing plants, especially customer influence on lead time (empirical evidence)	explicitly used	explicitly used
Oke (2003b)	inventory and volume flexibility within order fulfillment (empirical evidence)	explicitly used	explicitly used
Barad and Sapir (2003)	flexibility benefits in logistic systems (quantitative approach)	explicitly used	explicitly used
Takahashi and Nakamura (2000)	reactive Just-in-Time order systems to achieve agile control in the face of unstable demand characteristics (simulation study)	implicitly implied	explicitly used
Koste et al. (1999)	framework for manufacturing flexibility, using 10 flexibility dimensions	implicitly implied	implicitly implied

From the quantitative point of view there is also a lot of literature available overcoming the limitation of qualitative approaches which are often not able to operationally specify managerial implications and measure their outcomes. In order to motivate the two flexibility measures proposed above (the ability to handle a short customer required lead time and a certain level of customer demand variation) Tab. 1 provides an overview of relevant literature addressing flexibility issues within a quantitative context whereby column three and four, respectively, indicate whether a contribution applies/includes these two measures. The classification "explicitly used" reports a direct application of the respective measure in a research paper whereas "implicitly implied" means that arguments with a distinct relation to this measure are discussed, but it is not directly mentioned.

Tab. 1 shows that the capability of being able to cope with demand variations is explicitly mentioned as flexibility means in most of the quantitative contributions reviewed, whereas the mean customer required lead time flexibility is more often addressed indirectly. In this regard typical arguments indirectly addressing this topic have incorporated related measures such as a low utilization, low inventory levels or the capacity to quickly react in the face of changes – all of them implying the capability to operate with short lead times. A main conclusion is therefore, that the two initially proposed abilities to handle short mean customer required lead time and a certain level of customer demand variation are found to be reasonable measures for flexibility in an industrial Value Chain context, at the same time being compatible to the generic flexibility definitions discussed above.

2.2 Production logistical relationships concerning the defined flexibility measures

The relationship between WIP, utilization and production lead time has been discussed in a lot of publications based on queuing theory, see Little (1961), Medhi (1991), Chen and Yao (2001), Tijms (2003) and Hopp and Spearman (2008). Some other analytical models discussing this relationship are e.g. developed by Missbauer *et al.* (1998) and Jodlbauer (2008a). Additionally, there are a lot of empirical studies on this relationship available either applying simulation, see Jones (1973), Hopp and Roof-Sturgis (2000), Jodlbauer and Huber (2008), Altendorfer *et al.* (2007a), and Altendorfer *et al.* (2007b), or analyzing real company data, see Nyhuis and Wiendahl (2002), Wiendahl and Breithaupt (1999) and Wiendahl *et al.* (2005). In all these models the link to service level or tardiness is missing.

This link to service level and/or tardiness has recently been examined by Jodlbauer (2008b), Altendorfer and Jodlbauer (2011) and Jodlbauer and Altendorfer (2010) applying either queuing theory or other analytical methods. Based on the results of Jodlbauer (2008b) and Jodlbauer and Altendorfer (2010) the capacity needed to reach a capacity-related service level can be calculated based on the capacity demand fluctuation and the customer required lead time distribution. This approach, called customer driven production planning, is extended in the current paper to discuss the relationship between the two proposed flexibility measures.

3. Model development

3.1 Assumptions and Notation

We study a multi-item single-stage MTO production system with the structure shown in Fig. 1:

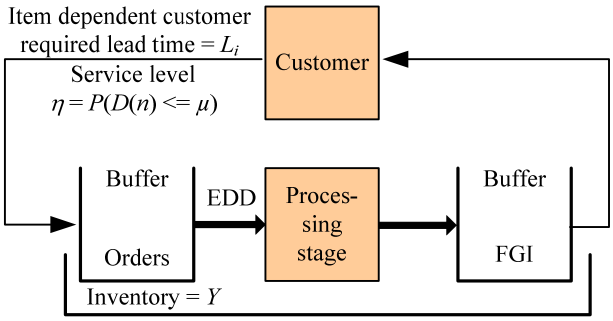


Fig. 1. Production system structure.

In this system, each item $i \in 1,...,N$ has a random customer demand in pieces $A_i(n)$ for an arbitrary time period *n*, which is independent of other item demands and which is independent of the previous time period demands. This $A_i(n)$ follows a Gauss Process (see Ross (2010) or Beichelt (2006) for details on Gauss Processes) with $E\left[A_i(n)\right] = \lambda_i n$ and $Var\left[A_i(n)\right] = \sigma_i^2 n$ which states that the demand for each item in any arbitrary time period *n* is normally distributed (see also Altendorfer (2010) for this formulation of the setting discussed in Jodlbauer and Altendorfer (2010)). λ_i and σ_i^2 denote the mean demand rate and the demand variance of item *i* respectively. Following Altendorfer (2010) with defining $\overline{\lambda} = \sum_{i=1}^{N} \lambda_i q_i$ and $\sigma^2 = \sum_{i=1}^{N} (\sigma_i q_i)^2$ (whereby q_i is the item *i* dependent deterministic capacity needed to produce one piece) the mean and variance of capacity demand can be identified as $E\left[D(n)\right] = \overline{\lambda} n$ and $Var\left[D(n)\right] = \sigma^2 n$. $f_{D(n)}(\cdot)$ and $F_{D(n)}(\cdot)$ denote the probability density function (pdf) and cumulated distribution function (cdf) of the random variable $D(n)$ respectively. Based on the order date and the random due date the customer required lead time $\overline{L}$ can be defined as the random customer required lead time of one capacity unit (one time unit needed from the machine). For a derivation of $\overline{L}$ depending on the item dependent customer required lead time values L_i see Altendorfer (2010). $f_{\bar{L}}(\cdot)$ and $F_{\bar{L}}(\cdot)$ denote the pdf and cdf of the random variable $\bar{L}$ respectively. The processing stage processes orders at a rate of μ.

Tab. 2. List of variables.

Variable	Description
$A_i(n)$	random customer demand in pieces for item *i*
$E[\cdot]$, $Var[\cdot]$	expected value and variance for a random variable respectively
λ_i, σ_i^2	mean demand rate and the demand variance of item i respectively
$\bar{\lambda}$, σ^2	mean and variance of capacity demand rate respectively
$D(n)$	random customer demand in capacity for time period n
$f_R(\cdot)$, $F_R(\cdot)$	probability density function (pdf) and cumulated distribution function (cdf) of the random variable R respectively
q_i	deterministic capacity needed to produce one piece of item i
L_i	random item i dependent customer required lead time
$\bar{L}$	random customer required lead time of one capacity unit
μ	processing rate of the single-stage
P	minimum required production lead time of processing stage
$\alpha_{D(n)}$	coefficient of variation of *D(n)*
s	service level
s_P	percentage of capacity which cannot be delivered on time (based on *P*)
n	length of capacity smoothing period
$E\left[\bar{L}_{\min}\right]$	minimum mean customer required lead time which can be provided to reach a certain service level s
X	work ahead window (parameter for work release policy)
Y	random inventory in capacity units
c_h	inventory holding cost factor
c_μ	capacity cost factor
$C(\cdot)$	overall costs for capacity and inventory

All orders are released to the production system according to a work ahead window strategy (see Jodlbauer and Altendorfer (2010) or Altendorfer and Jodlbauer (2011)) and are processed according to the earliest due date (EDD) dispatching rule. In front of the processing stage, the released materials are stored in a buffer and all orders finished prior to the due date, are stored in the finished-goods-inventory. A capacity service level *s*, stating what percentage of capacity is delivered on time, can be calculated. Jodlbauer and Altendorfer (2010) argue that in such a system the capacity service level can be applied to approximate the often used β-service level and is equal to this β-service level whenever processing time is equal for each item. The production system is modeled as having a minimum required production lead time *P*. This value is needed to include technical constraints as well as transportation times or organizational times before production. Tab. 2 provides a list of variables.

In this setting three propositions concerning the minimum mean customer required lead time and the optimal costs with respect to a predefined capacity demand variation are stated. Firstly, for a system with predefined capacity it is shown what the lower bound of mean customer required lead time is. Secondly, the problem is extended to a setting where the capacity investment is optimized. In this second setting again a lower bound for mean customer required lead time and the cost minimum are provided. Mean customer required lead time and optimal costs are both linked to the capacity demand variation showing its influence within a Value Chain setting with a downstream customer and its upstream supplier.

3.2 Relationship between the two flexibility measures

Following the concept of Jodlbauer (2008b), Jodlbauer and Altendorfer (2010) and Altendorfer (2010) it is obvious that the random capacity demand $D(n)$ will, for some time periods *n*, be greater than the available capacity μn in this time period. Any time period *n* can be interpreted as an averaging period for smoothing capacity demand since the coefficient of variation of $D(n)$ decreases with increasing *n*:

$$\alpha_{D(n)} = \frac{\sqrt{Var\left[D(n)\right]}}{E\left[D(n)\right]} = \frac{\sigma}{\bar{\lambda}\sqrt{n}} \Rightarrow \frac{d\alpha_{D(n)}}{dn} = -\frac{1}{2}\frac{\sigma}{\bar{\lambda}}n^{-\frac{3}{2}} \quad \textbf{(1)}$$

According to Altendorfer (2010), there is always a certain percentage of capacity which cannot be delivered on time for orders with customer required lead time shorter than *P* (see also service level definition in Section 3.1):

$$s_P = F_{\bar{L}}\left(P\right) \quad \textbf{(2)}$$

Based on equation (1) and (2), the following service level equation shows that for a predefined capacity μ, the service level reached increases in the capacity smoothing period applied:

$$F_{D(n)}\left(\mu n\right) - s_P = s \quad \textbf{(3)}$$

Following Jodlbauer and Altendorfer (2010) and Altendorfer (2010) shows that for normally distributed demand, the following smoothing period is needed in order to reach a predefined service level *s*:

$$n = \left(\frac{F^{-1}_{N(0,1)}\left(s + s_P\right)}{\mu - \bar{\lambda}}\right)^2 \sigma^2 \quad \textbf{(4)}$$

whereby $F_{N(0,1)}^{-1}(\cdot)$ denotes the quantile (inverse) of the standard normal distribution function.

From Altendorfer (2010), the following relationship concerning the customer required lead time distribution has to hold, to ensure that a certain service level can be reached in the MTO setting:

$$E\left[\bar{L}\right]-\int_0^P 1-F_{\bar{L}}(\tau)d\tau > n(\mu,s) \tag{5}$$

For low *P* values ($P \ll E\left[\bar{L}\right]$) the following approximation is stated in Altendorfer (2010):

$$\int_0^P \left(1-F_{\bar{L}}(\theta)\right)d\theta \approx P \tag{6}$$

Furthermore, for low *P* values ($P \ll E\left[\bar{L}\right]$), the service level loss s_P is very low and is therefore neglected:

$$s_P = F_{\bar{L}}\left(P\right) \text{ with } F_{\bar{L}}\left(P\right) \ll 1 \Rightarrow s_P \approx 0 \tag{7}$$

Proposition 1:

The minimum mean customer required lead time $E\left[\bar{L}_{\min}\right]$ *which is still able to handle a predefined capacity demand variation* σ^2 *under normally distributed demand at a single-stage production system with respect to a predefined service level s is defined as (applying approximation (6) and (7)):*

$$E\left[\bar{L}_{\min}\right]=\left(\frac{F_{N(0,1)}^{-1}(s)}{\mu-\bar{\lambda}}\right)^2\sigma^2+P \tag{8}$$

Proof:

Proposition 1 follows directly from equation (4) to (7) with:

$$E\left[\bar{L}\right]-\int_0^P 1-F_{\bar{L}}(\tau)d\tau > n(\mu,s) \to \min_{E\left[\bar{L}\right]}$$
$$\Leftrightarrow E\left[\bar{L}_{\min}\right]=n(\mu,s)+P \tag{9}$$

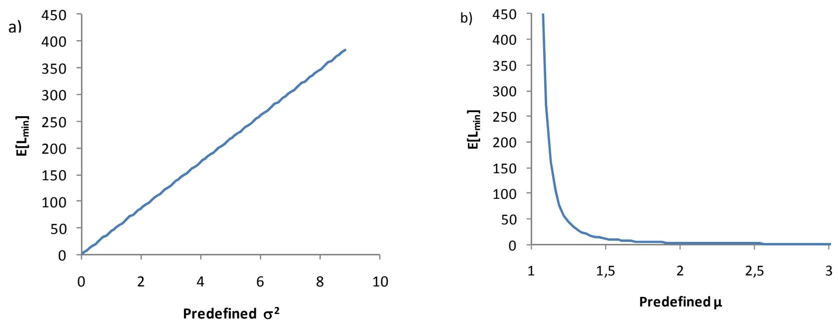
Proposition 1 states the relationship between customer required lead time flexibility and capacity demand variance flexibility with respect to the capacity available. Note that Proposition 1 is similar to the condition stated as MTO ability of a production system in Jodlbauer (2008b), however the relationship is stated there as being an inequality. The minimum mean customer required lead time $E\left[\bar{L}_{\min}\right]$

has a linear increase in the capacity demand variance and a convex decrease in the capacity available, see:

$$\frac{dE\left[\bar{L}_{\min}\right]}{d\sigma^2}=\left(\frac{F^{-1}_{N(0,1)}(s)}{\mu-\bar{\lambda}}\right)^2>0 \text{ and } \frac{dE\left[\bar{L}_{\min}\right]}{d\mu}=-2\frac{\left(F^{-1}_{N(0,1)}(s)\right)^2}{\left(\mu-\bar{\lambda}\right)^3}\sigma^2<0 \tag{10}$$

$$\frac{dE\left[\bar{L}_{\min}\right]}{d^2\mu}=6\frac{\left(F^{-1}_{N(0,1)}(s)\right)^2}{\left(\mu-\bar{\lambda}\right)^4}\sigma^2>0$$

This holds since $\mu>\bar{\lambda}$ is assumed for a stable system with utilization below 100%. Fig. 2 visualizes this relationship:



$P=1;\ s=0.95;\ \bar{\lambda}=1;\ \mu_{\text{Case a)}}=1.25;\ \sigma^2_{\text{Case b)}}=1$

Fig. 2. $E\left[\bar{L}_{\min}\right]$ with respect to σ^2 and μ.

An interesting finding from Proposition 1 is that the variance of customer required lead time has no influence on the minimum mean customer required lead time which can be maintained when applying approximations (6) and (7). However, when the mean customer required lead time is higher than the minimum value, the work ahead window X for planning is implicitly defined as (see Altendorfer (2010)):

$$\int_P^X\left(1-F_{\bar{L}}(\theta)\right)d\theta=n \tag{11}$$

which shows that in this situation the distribution of customer required lead time is also needed. This holds since the work ahead window is ∞ when calculating $E\left[\bar{L}_{\min}\right]$.

3.3 Optimal capacity investment

Before optimizing the capacity investment, the inventory calculation from Jodlbauer and Altendorfer (2010) and Altendorfer (2010) is introduced briefly. Based on the minimum required production lead time *P* and on the work ahead window work release policy with parameter *X*, see equation (11), the inventory follows as (for a derivation see Altendorfer (2010)):

$$E\left[Y\right]=\left(P+n\left(\mu,s\right)\right)\bar{\lambda} \tag{12}$$

Term one of (12) corresponds to the inventory needed to maintain the minimum required lead time of the system and term two is defined as surplus inventory and corresponds to the inventory from pre-producing on stock.

When minimizing the sum of inventory holding cost and capacity investment cost under a service level constraint $\tilde{s}$ (problem (13)), Altendorfer (2010) proves the following optimal capacity investment and inventory (see (14)) under the condition (15) (applying approximations (6) and (7)):

$$C\left(\mu,X\right)=E\left[Y\right]c_h+\mu c_\mu \rightarrow \min_{\mu,X} \tag{13}$$

w.r.t. $s\geq\tilde{s}$

$$\mu^*=\left(2\bar{\lambda}c_h\frac{\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^2\sigma^2}{c_\mu}\right)^{\frac{1}{3}}+\bar{\lambda} \tag{14}$$

$$E\left[Y^*\right]=\left(P+\left(\frac{F_{N(0,1)}^{-1}\left(\tilde{s}\right)\sigma c_\mu}{2\bar{\lambda}c_h}\right)^{\frac{2}{3}}\right)\bar{\lambda}$$

$$n\left(\mu^*,\tilde{s}\right)=\left(\frac{F_{N(0,1)}^{-1}\left(\tilde{s}\right)\sigma c_\mu}{2\bar{\lambda}c_h}\right)^{\frac{2}{3}}$$

$$E\left[\bar{L}\right]-P>n \tag{15}$$

Proposition 2:

The minimum mean customer required lead time which is still able to handle a predefined capacity demand variation σ^2 *under normally distributed demand at a single-stage production system when optimizing capacity and inventory costs with respect to a service level constraint* $\tilde{s}$ *is defined as:*

$$E\left[\bar{L}_{\min}\right]=\left(\frac{F_{N(0,1)}^{-1}\left(\tilde{s}\right)\sigma c_\mu}{2\bar{\lambda}c_h}\right)^{\frac{2}{3}}+P \tag{16}$$

and $E\left[\bar{L}_{\min}\right]$ *has a concave increase with respect to the capacity demand variance* σ^2.

Proof:

Equation (16) follows directly from equation (8) and (14); and

$$\frac{dE\left[\bar{L}_{\min}\right]}{d\sigma^2}=\frac{1}{3}\left(\sigma^2\right)^{-\frac{2}{3}}\left(\frac{F_{N(0,1)}^{-1}\left(\tilde{s}\right)c_\mu}{2\bar{\lambda}c_h}\right)^{\frac{2}{3}}>0 \qquad \textbf{(17)}$$

$$\frac{dE\left[\bar{L}_{\min}\right]}{d^2\sigma^2}=-\frac{2}{9}\left(\sigma^2\right)^{-\frac{5}{3}}\frac{\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)c_\mu\right)^{\frac{2}{3}}}{\left(2\bar{\lambda}c_h\right)^{\frac{2}{3}}}<0$$

shows the concave increase.

Note that this relationship only states what the minimum mean customer required lead time has to be when capacity investment can be optimized and capacity demand variance is predefined.

Comparing equation (8) with equation (16) shows that in the cost optimal case, the increase of $E\left[\bar{L}_{\min}\right]$ with respect to σ^2 is no longer independent of σ^2. Taking equation (14) into account shows that an increase in σ^2 also leads to an increase in optimal capacity to invest.

Proposition 3:

The optimal cost for capacity and inventory for a single-stage production under normally distributed demand with respect to a service level constraint follows as:

$$C^*\left(\sigma^2\right)=\left(\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^2\sigma^2 c_\mu{}^2\bar{\lambda}c_h\right)^{\frac{1}{3}}\left(2^{-\frac{2}{3}}+2^{\frac{1}{3}}\right)+\left(Pc_h+c_\mu\right)\bar{\lambda} \qquad \textbf{(18)}$$

which is a concave increasing function in capacity demand variance.

Proof :

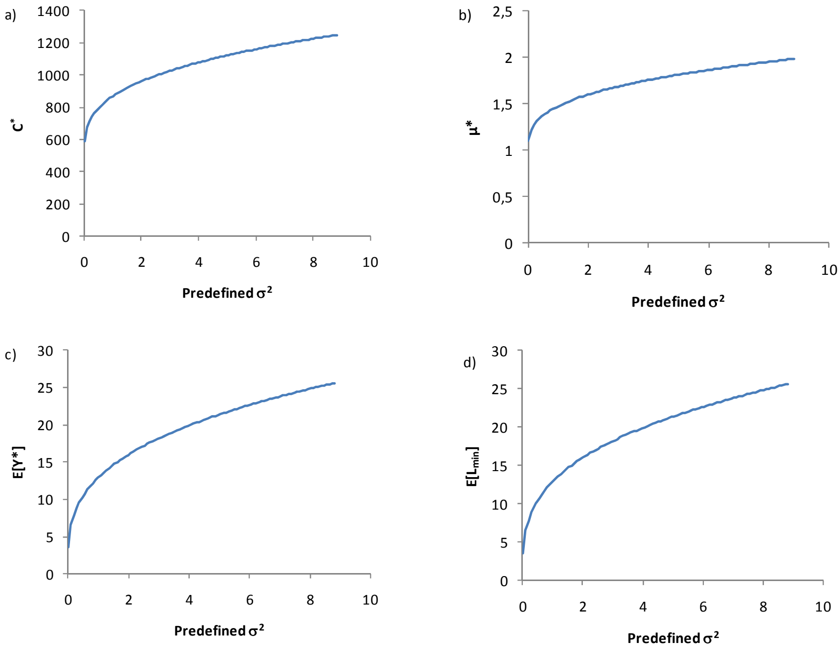
Equation (18) follows from equation (13) and (14):

$$C^*\left(\sigma^2\right)=\left(P+\left(\frac{F_{N(0,1)}^{-1}\left(\tilde{s}\right)\sigma c_\mu}{2\bar{\lambda}c_h}\right)^{\frac{2}{3}}\right)\bar{\lambda}c_h+\left(\left(\frac{2\bar{\lambda}c_h\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^2\sigma^2}{c_\mu}\right)^{\frac{1}{3}}+\bar{\lambda}\right)c_\mu \qquad \textbf{(19)}$$

$$=\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^{\frac{2}{3}}\sigma^{\frac{2}{3}}c_\mu^{\frac{2}{3}}\bar{\lambda}^{\frac{1}{3}}c_h^{\frac{1}{3}}2^{-\frac{2}{3}}+\bar{\lambda}^{\frac{1}{3}}c_h^{\frac{1}{3}}\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^{\frac{2}{3}}\sigma^{\frac{2}{3}}c_\mu^{\frac{2}{3}}2^{\frac{1}{3}}+\left(Pc_h+c_\mu\right)\bar{\lambda}$$

The concave increase follows from the first two derivatives:

$$\frac{dC^*\left(\sigma^2\right)}{d\sigma^2}=\frac{1}{3}\left(\sigma^2\right)^{-\frac{2}{3}}\left(\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^2 c_\mu{}^2\bar{\lambda}c_h\right)^{\frac{1}{3}}\left(2^{-\frac{2}{3}}+2^{\frac{1}{3}}\right)>0$$
$$\frac{dC^*\left(\sigma^2\right)}{d^2\sigma^2}=-\frac{2}{9}\left(\sigma^2\right)^{-\frac{5}{3}}\left(\left(F_{N(0,1)}^{-1}\left(\tilde{s}\right)\right)^2 c_\mu{}^2\bar{\lambda}c_h\right)^{\frac{1}{3}}\left(2^{-\frac{2}{3}}+2^{\frac{1}{3}}\right)<0 \quad \textbf{(20)}$$



$P=1;\ \tilde{s}=0.95;\ \bar{\lambda}=1;\ c_\mu=500;\ c_h=10$

Fig. 3. Optimal cost and flexibility with capacity and inventory costs.

For practical application, equation (16) shows the minimum mean customer required lead time which can be handled for a predefined capacity demand variance and a predefined service level constraint. Furthermore, the influence of reducing the capacity demand variance on minimum mean customer required lead time which can be handled and on optimal costs is shown. From equation (20) it becomes clear that even if the optimal costs can be reached with the occurring mean customer required lead time, a higher capacity demand variance leads to higher costs.

Fig. 3 shows the optimal costs, the minimum customer required lead time to reach them, the optimal capacity investment and the optimal inventory with respect to capacity demand variance.

3.4 Additional costs for lower mean customer required lead time

In this subsection the additional costs for a shorter mean customer required lead time than calculated in equation (16) are evaluated.

Based on the optimal costs from equation (18) which only hold if $E\left[\bar{L}\right] > E\left[\bar{L}_{\min}\right]$, the additional costs for $E\left[\bar{L}\right] < E\left[\bar{L}_{\min}\right]$ are calculated. In this case, constraint (15) of the optimization problem is binding and therefore n is defined as:

$$n = E\left[\bar{L}\right] - P \tag{21}$$

With this new n, the capacity to invest μ follows from equation (4) and (7) as:

$$E\left[\bar{L}\right] - P = \left(\frac{F_{N(0,1)}^{-1}(\tilde{s})}{\mu - \bar{\lambda}}\right)^2 \sigma^2 \Leftrightarrow \mu = \bar{\lambda} + \frac{F_{N(0,1)}^{-1}(\tilde{s})\sigma}{\sqrt{E\left[\bar{L}\right] - P}} \tag{22}$$

and the expected inventory can also be calculated based on the new n as (see equation (12)):

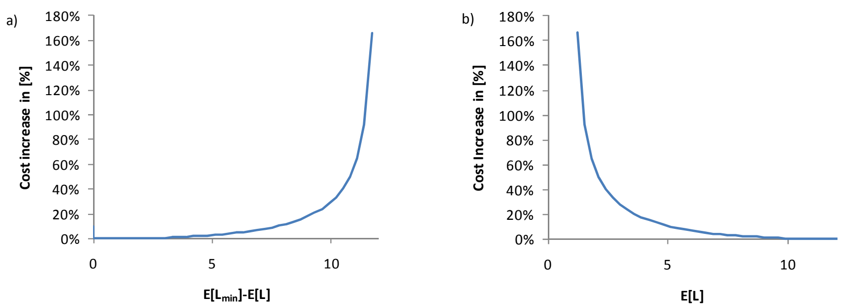
$$E[Y] = \left(E\left[\bar{L}\right] - P + P\right)\bar{\lambda} = E\left[\bar{L}\right]\bar{\lambda} \tag{23}$$

which leads to costs of:

$$C\left(\sigma^2, E\left[\bar{L}\right]\right) = E[Y]c_h + \mu c_\mu = E\left[\bar{L}\right]\bar{\lambda}c_h + \left(\bar{\lambda} + \frac{F_{N(0,1)}^{-1}(\tilde{s})\sigma}{\sqrt{E\left[\bar{L}\right] - P}}\right)c_\mu \tag{24}$$

This cost from equation (24) can be compared to the optimal cost from (18) and provides a manager with the information how expensive it is to maintain the lower mean customer required lead time.

The following Fig. 4 shows this additional cost with respect to $E\left[\bar{L}_{\min}\right] - E\left[\bar{L}\right]$.



$P = 1;\ \tilde{s} = 0.95;\ \bar{\lambda} = 1;\ \sigma^2 = 1;\ E\left[\bar{L}_{\min}\right] = 12.9$

Fig. 4. Cost increase when $E\left[\bar{L}\right] < E\left[\bar{L}_{\min}\right]$.

From a Value Chain perspective this additional cost shows the potential cost benefit of improving information flow, which leads to an increased mean customer required lead time $E\left[\bar{L}\right]$ since orders are stated earlier. The cost reduction potential of decreasing $E\left[\bar{L}_{\min}\right]-E\left[\bar{L}\right]$ can, for example, be balanced against the additional cost of information technology improving the information flow.

4 Numerical study

This section discusses the influence of the predefined service level constraint, the minimum required production lead time and the capacity and inventory costs on the relationship between $E\left[\bar{L}_{\min}\right]$ and σ^2. In subsection 4.1 the curves shown in Fig. 3 are compared for different P, $\tilde{s}$, c_μ and c_h values. In subsection 4.2 a sensitivity analysis showing $E\left[\bar{L}_{\min}\right]$ with respect to P, $\tilde{s}$, c_μ and c_h for a predefined σ^2 is provided. In this section the cost optimal $E\left[\bar{L}_{\min}\right]$ from equation (16) is discussed.

4.1 Parameter comparison examples

To show the influence of different parameter sets on the relationship between $E\left[\bar{L}_{\min}\right]$ and σ^2, the following examples are compared for the setting in which capacity and inventory costs are minimized:

- Example 1 – basic scenario: $P=1;\ \tilde{s}=0.95;\ \bar{\lambda}=1;\ c_\mu=500;\ c_h=10$
- Example 2 – production lead time: $P=2;\ \tilde{s}=0.95;\ \bar{\lambda}=1;\ c_\mu=500;\ c_h=10$
- Example 3 – service level: $P=1;\ \tilde{s}=0.90;\ \bar{\lambda}=1;\ c_\mu=500;\ c_h=10$
- Example 4 – capacity costs: $P=1;\ \tilde{s}=0.95;\ \bar{\lambda}=1;\ c_\mu=600;\ c_h=10$
- Example 5 – inventory holding costs: $P=1;\ \tilde{s}=0.95;\ \bar{\lambda}=1;\ c_\mu=500;\ c_h=20$

Fig. 5 shows the results of this study. Comparing the results of example 1 with example 2 shows that an increase in minimum production lead time has only a low influence on the optimal capacity to invest and it leads to a slight cost increase as well as to a slight increase in $E\left[\bar{L}_{\min}\right]$ which means to a slight flexibility loss. Looking at example 3 provides the insight that a decrease of the service level constraint decreases costs, optimal capacity, optimal inventory and $E\left[\bar{L}_{\min}\right]$. Therefore, accepting a lower service level from the upstream supplier in a Value Chain can improve this supplier's flexibility concerning $E\left[\bar{L}_{\min}\right]$ and σ^2. The comparison of examples 4 and 5 with example 1 shows an interesting difference in the effect of cost increases. While a cost increase in capacity costs leads to a worse situation for all parameters (costs are higher and flexibility decreases), the cost increase in inventory holding costs shows that in this case the optimal costs increase, however, the flexibility also increases (via a decreasing $E\left[\bar{L}_{\min}\right]$).

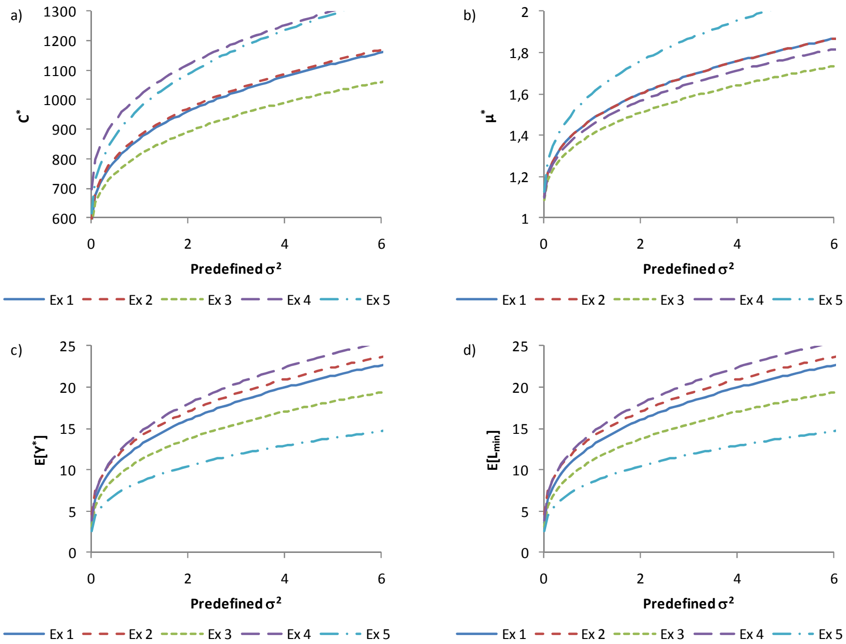


Fig. 5. Numerical study – exemplary comparison.

4.2 Numerical sensitivity analysis

To study the effect of the parameters P, $\tilde{s}$, c_μ and c_h more in detail, in this subsection a numerical sensitivity analysis is provided. The coefficient of variation of capacity demand is therefore predefined with 1 and the parameters are varied in certain ranges. All parameters, except the one varied, are set to the values of example 1 from subsection 4.1.

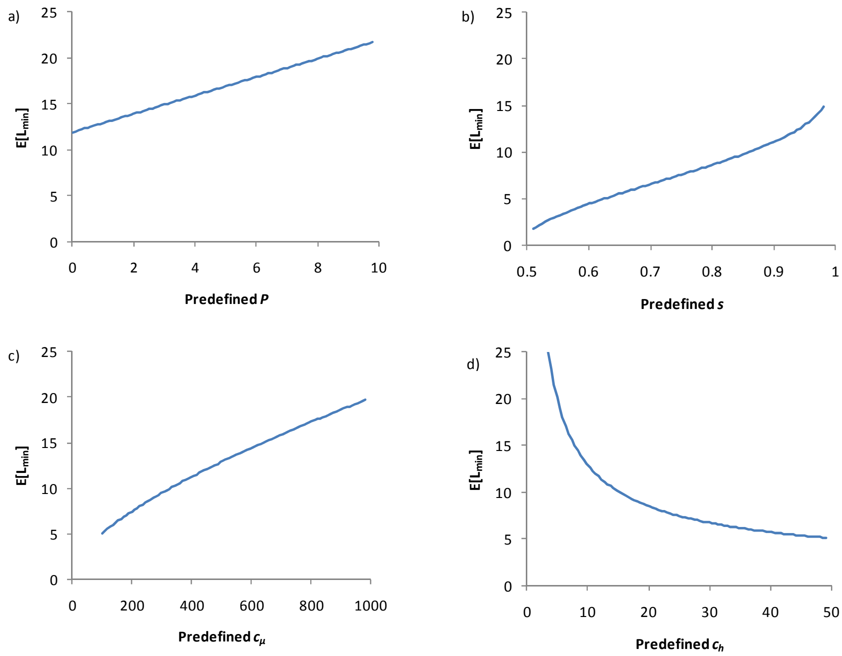


Fig. 6. Numerical study – sensitivity analysis.

Fig. 6 shows that customer required lead time flexibility decreases when minimum production lead time, capacity costs and service level increase. However, with higher inventory cost, there is a rise of optimal customer required lead time flexibility. One managerial insight from this analysis can be derived from the curve shape of in Fig. 6b) which shows that especially service level constraints above 95% lead to a mayor decrease in flexibility. Another managerial insight can be found in Fig. 6d) which shows that in the cost optimal setting, higher valued products (with higher inventory holding costs) already imply that a higher customer required lead time flexibility is optimal. Referring to the initially mentioned matter of Value Chain impacts resulting from local production decisions and performance states (as well as the possible amplification effects across the entire value creation process), it is important for responsible production managers to capture a better understanding of local sensitivities regarding their impact on overall Value Chain behavior.

5. Conclusion

In this paper the capability to handle a certain capacity demand variance and a certain mean customer required lead time is assumed to be a possible measure for flexibility in the relationship between a downstream company and its upstream suppliers within a Value Chain. For a predefined capacity and service level it is found that capacity demand variance and mean customer required lead time are not direct substitutes. With respect to a service level constraint, predefining a certain capacity variance demanded by the customer(s) only leads to a lower bound of mean customer required lead time which still can be provided. Optimizing the capacity investment and inventory costs at the supplier site with respect to a service level constraint and a predefined capacity variance again leads to a certain lower bound for the mean customer required lead time which can still be maintained. Further, the additional costs arising from situations with a customer required lead time below this value are discussed. For practical application this paper provides the managerial insight that the lower bound of the mean customer required lead time flexibility is a function of the customer demand variance and the (optimal) capacity available. Applying the findings of this study to a Value Chain shows that cost benefits can be achieved by measures for smoothing the capacity demand variance and by measures for improving the information flow (which leads to an increased mean customer required lead time) within the Value Chain.

We recommend future research on the extension of the presented method, especially regarding optimization of capacity demand variance and customer required lead time as decision variable between customers and suppliers within a Value Chain.

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